Optimal Taxation with Misperception on Prices

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Abstract

Optimal linear commodity tax mixed with non-linear labor income tax formulas are explored when inattentive agents misperceive prices and marginal income tax rates. Both mechanism design and tax perturbation method are used to express optimal tax formulas in measurable sufficient elasticities and misperception wedges. We derive modified IC constraint, and highlight the role of indirect taxation in correcting commodity price perception wedge and helping government to get a more progressive distribution. Optimal income tax is verified to take the modified form of Diamond's ABC-formula, and needs to correct both misperception wedge of income tax and externalities of marginal tax rate at one income level on tax perception at other income levels. We find that misperception causes several modifications on optimal tax rule with rational agents. Firstly, traditional many-person Ramsey rule is modified by rescaling the covariance term and by adding bias-correcting terms. Secondly, uniform tax rule fails under typical preference structure for Atkinson and Stiglitz theorem. Within group uniform tax rule is also hampered by misperception. Another verification is the connection between Corlett-Hague rule and many person Ramsey rule in the mixed taxation environment by proving the equivalence of results from mechanism design and tax perturbation method.

Keywords: Misperception on prices; Optimal non-linear income taxation; Optimal linear direct taxation

JEL Classification: D61, H21, H23

I. Introduction

Aspects from psychology is quite important in explaining departures from standard competitive, general equilibrium models in the real world, since human beings can hardly meet fully rational assumptions. Only after comprehensively understanding the decisions by behavioral agents, it is

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possible for policy makers to review the effects of public policies, assess possible behavioral wedges, and adjust future policies to accommodate reactions of behavioral agents.

Imperfect perception on prices or tax rates. Misperception on prices or tax rates is quite typical in empirical evidence of bounded rationality. For commodity taxes, Chetty, Looney and Kroft (2009) find that consumers make systematic optimization errors with respect to commodity taxes since these taxes are not fully salient through a field experiment. Taubinsky and Rees-Jones (2017) use an online shopping experiment to prove heterogeneity among consumer's under-reaction to not-fully-salient commodity taxes, and analyze the welfare loss of taxation. For income tax, Liebman and Zeckhauser (2004) and Rees-Jones and Taubinsky (2019) argue that consumers generally use ironing heuristic to understand complex income tax schedule.

Then how to adjust tax policy according to those inattentive consumer's behavior? Or a step further, how would traditional tax rule with rational agents modified when people misperceive prices and taxes? Most relevant research concentrates on adjustments of single tax under misperception, like Farhi and Gabaix (2020). What remains to be explored is the design of optimal mixed taxation with behavioral agents. Since consumers' behavioral bias (i.e. misperception on marginal labor income tax rate or commodity prices) affects both labor supply and consumption, in the context of mixed taxation, optimal linear commodity tax may be influenced by misperception of income tax, and vice versa. This paper revisits optimal linear commodity tax mixed with non-linear income tax when inattentive agents misperceive prices and marginal income tax rates, and expresses optimal tax formulas in terms of sufficient statistics. Modification on traditional tax rules are then examined.

Main results on the design of mixed taxation. We follow the approach of mechanism design in Mirrlees (1976) to solve government's optimal control problem and our first contribution is to demonstrate two aspects through which misperception influences the first-order incentive constraint. Firstly, misperception on marginal labor income tax directly modifies the expression of first-order incentive constraint. For instance, if consumer perceives a higher marginal tax rate, he would be more likely to mimic labor supply of individuals with lower ability. Secondly, misperception of both tax schedule modifies the impact of commodity price on first-order incentive constraint. In other words, it changes the redistributive role of commodity tax. Specifically, actual commodity prices affect commodity price perception wedge which correlates with labor supply, while income tax perception would amplify this correlation as well as correlation between commodity demands and labor supply. Actual commodity price also directly alters consumer's perception of marginal income tax rate by our assumption and then relaxes (or tightens) the first-order incentive constraints.

We then solve the optimal mixed taxation problem and characterize optimal tax system in measurable sufficient elasticities and perception wedges. Our optimal income tax is a combination of results in Jacobs and Boadway (2014), which explore optimal mixed taxation with fully rational individuals, and results in Farhi and Gabaix (2020), which derive optimal income tax formula with inattentive agents. Therefore, optimal marginal income tax rate for a behavioral individual earning labor income z depends not only on social marginal welfare weights, income distribution, elasticities of labor income and commodity demands, but also on his misperception on marginal income tax rate and externalities of marginal tax rate at z which influence misperception of income tax rate at other income levels. The influence of misperception on marginal income tax is found to be the same as in Farhi and Gabaix (2020) though we adopt a different analytical method. In our mixed taxation environment, regardless of impact of misperception on elasticities, demands and other endogenous variables, misperception on one tax instrument is directly corrected by the tax instrument itself. What is new in this paper is that due to misperception, commodity tax not only has the role in correcting the inefficiency among commodity demands resulted from misperception on commodity price, but also helps government to get a more progressive distribution since IC constraint is influenced by commodity price. This finding acts as our second contribution.

Applications: comparison with traditional tax rules. The two roles of commodity tax are instructive when we revisit traditional commodity tax rule. Even with the typical preference structure for Atkinson and Stiglitz theorem, linear commodity tax may not be superfluous when misperception exists. Differentiated commodity tax policy, seemingly discretionary with rational agents, not only helps to reduce inefficiency among commodity demands caused by misperception on commodity prices, but also utilize re-distributive role of indirect tax through influence of commodity price on perceived marginal income tax. As for within group uniform tax rule, goods within subgroup utility function can not project to a composite good since we cannot extract a homogeneous price of that virtual good with misperception on prices. Therefore, uniform subgroup commodity tax no longer applies.

Main analytical method. Our model is based upon inattention modeling framework in

Gabaix (2014), which captures a wide range of behavioral phenomena such as inattention to prices, nominal illusion, hyperbolic discounting, et al. Compared with other models, Gabaix's model is more tractable and fairly unified as it adapts to both microeconomic problems like basic consumer theory and Arrow-Debreu-style general equilibrium (Gabaix 2014), dynamic macroeconomics (Gabaix 2016) and public economics (Farhi and Gabaix 2020). To link consumer's inattentive behavior with rational choices, we find as-if rational consumers who make the same consumption and labor supplying decisions under perceived prices.

We mainly follow Mirrlees (1971), Mirrlees (1976) and Jacobs and Boadway (2014) by taking their mechanism design approach to get optimal tax formula. We also use tax perturbation method in Saez (2001) and Farhi and Gabaix (2020) to compare the results. Then our third contribution comes as we connect many person Ramsey rule with Corlett-Hague rule by proving the equivalence of two optimal commodity formulas derived from mechanism design approach and tax perturbation method.

Related literature. Our work relates to two subfields in optimal taxation theory. The first is optimal taxation with inattentive agents. Misperception is a typical phenomenon of inattention since many actual prices or tax rates are far from salient. Chetty, Looney and Kroft (2009) propose an approach to compute welfare and efficient loss due to salient effects. Liebman and Zeckhauser (2004) solve optimal income tax when consumers observe nonlinear income taxation with ironing heuristic. In a tractable and general framework modeling inattention, Farhi and Gabaix (2020) use the difference between actual prices and marginal utility vectors expressed in a money metric to define behavioral wedges, and then update optimal taxation formula. Boccanfuso and Ferey (2019) and Moore and Slemrod (2020) address the endogeneity of taxpayers' attention. The former research captures how information frictions in tax perceptions affect the design of optimal income tax. The later one models how non-rate policy instruments like nudge may change both taxpayer incentives and biases.

However, limited work has been done on how misperception of different tax instruments might shape optimal tax schedule in a mixed taxation environment. Actually, this problem is of realistic significance since the tax categories in the real world are manifold and tax systems are notoriously complex. To gain a full understanding of welfare impacts of certain misperception wedge, we need to analyze the cross-influence of misperception of one kind of tax on the design of another kind of tax. In this paper, we extend the literature by capturing such influence in a tax system with linear commodity tax and nonlinear income tax and clarifying the roles of both tax instruments.

This paper also contributes to the literature on optimal mixed taxation or the discussion about direct/indirect tax problem. There is a rich literature about optimal commodity taxes mixed with non-linear income taxes with fully-rational agents, and the role of indirect tax is widely discussed. One cornerstone is the Atkinson-Stiglitz theorem, which states that only non-linear income taxation is required to reach the optimum (Atkinson and Stiglitz 1976). Later discussion on uniform tax rule could be found in Deaton (1979), Besley and Jewitt (1995) and so on. There are also papers emphasizing the role of indirect taxation. Mirrlees (1976) finds out that commodities taxes should be greater on goods that are preferred by people with high ability. Christiansen (1984) points out that commodity should be taxed if it is positively related to leisure. Jacobs and Boadway (2014) argue that government should tax/subsidy commodities if they are more/less complementary with leisure than numerate good. Some literature focus on indirect taxation on a specific kind of good. For example, optimal inflationary tax mixed with non-linear income tax in da Costa and Werning (2008), capital tax in Golosov et al. (2013) and sin tax in Allcott, Lockwood and Taubinsky (2019). The former paper supports Friedman rule while the later two imply that indirect tax could be useful when consumers have heterogeneous preference, as has been discussed in Saez (2002).

Based on previous research, our work redefines the role of indirect tax by introducing misperception in this setting. We notice that commodity tax rule with misperception has just draw some attention in literature. For example, Allcott, Lockwood and Taubinsky (2018) find that if consumers are inattentive to commodity taxes when making labor supply decisions, optimal commodity tax schedule should follow the classic "many person Ramsey rule". By contrast, our framework differs from theirs as we keep the assumption in Chetty, Looney and Kroft (2009) and Taubinsky and Rees-Jones (2017) that consumers misperceive commodity tax rate or after-tax prices at the time of purchase. Therefore, while the scaling factor in Allcott, Lockwood and Taubinsky (2018) corresponds to misperception wedge when people making labor supply choices and captures the extent to which consumers' labor supply is sensitive to actual value of misperceived price, our model incorporates behavioral wedge when people make commodity purchasing decisions, so that rescaling effects of misperception wedge still exist if we preclude sensitivity of labor supply on prices.

Layout. The remainder of the paper is organized as follows. Section II defines individual's optimization problem and summarize important features of people's behavior with misperception on both linear commodity tax and non-linear labor income tax. Modified incentive constraint is presented and government's problem is defined in section III. Section V and IV derive optimal commodity tax and income tax in mixed tax schedule separately. Section VI provides an example with two goods. Section VII concludes. Omitted proofs are gathered in the appendix.

II. Individual Behavior under Misperceived Prices

In this section, we describe individuals' choices when they misperceive government's mixed tax schedule. We first set up the model by adding misperception into the framework of Atkinson and Stiglitz (1976). Then we disaggregate individual optimization into two stages as in Mirrlees (1976) and Jacobs and Broadway (2014). Assume that there is a continuum of consumers indexed by skill $n \in N \equiv [n_{min}, n_{max}]$ with density $f(n)$. The cumulative distribution function of skill is denoted by $F(n)$. The higher n, the more income could a consumer receive per unit of labor supplied. Consumer's utility depends on numerate good consumption c_n, general goods consumption vector x_n and labor supply l_n. Assume there are I kinds of general goods so that $x_n = (x_{1n}, x_{2n}, \ldots, x_{In})$. The price vector on general goods is $q = (q_1, q_2, \ldots, q_I)$. Consumers' preferences are homogeneous. Since consumer's before-tax labor income z_n depends on labor supply l_n and skill n , the utility function of a consumer with skill n could be written as

$$u(c_n, x_n, z_n/n), \forall n \in N. \tag{1}$$

The partial derivatives of utility take the following signs: $u_c(\cdot) > 0, u_{x_i}(\cdot) > 0, u_l(\cdot) < 0$. Let nonlinear tax function on consumer's income be denoted by $T(z_n)$, then consumer n's disposable income is $y_n \equiv z_n - T(z_n)$ and his marginal income tax rate is $Q_n = 1 - T'(z_n)$. However, consumers are not rational enough to fully perceive his marginal tax rate. The perceived marginal tax rate of consumer n is Q_n^s, which is influenced by actual marginal tax rate Q_n, actual tax schedule $\mathbf{Q}$, and actual general commodity price vector q. For simplicity, we normalize pre-tax prices of all commodities to unity so that the tax rate on commodity i is $t_i = q_i - 1$. Numerate good consumption is untaxed. In contrast to misperception on income tax, consumers also misperceive

q_i as $q_i^s(q)$. Overall, a consumer with skill n will maximize utility in (1) under budget constraint $c_n + \sum_i q_i x_{in} = z_n - T(z_n)$ and perceived prices $q^s(q)$ and $Q_n^s(q, Q_n, \mathbf{Q})$.

Next, we follow Jacobs and Broadway (2014) and Mirrlees (1976) and disaggregate consumers' optimization problem into two-stages. In the first stage, consumer with skill n chooses labor income z_n given real non-linear income tax schedule $T(z)$ and perception on income tax as $T^s(z_n)$. Consumer's choice in this stage determines his after-tax income to be $y_n = z_n - T(z_n)$. In the second stage, consumer chooses commodity consumption c_n and x_n given real price of commodities q, disposable income y_n, labor supply l_n and his perception of prices q^s.

A. Individual Optimization: Stage 2

We begin with the second-stage problem for consumer n. In the second stage, consumer misperceives commodity prices q_i as $q_i^s(q)$ and performs a sparse-max optimization as in Gabaix (2014):

$$\text{smax}_{c_n, x_n} u\left(c_n, x_n, z_n/n\right), s.t. c_n + \sum_i q_i x_{in} = y_n.$$

Consumer n chooses his consumption bundle (c_n, x_n) taking as given his labor supply z_n/n and disposable income y_n. The *smax* operator indicates marginal substitution rates are determined by relative observed prices, while consumer's actual consumption level is restricted by his budget constraint. We adopt a general form of q^s, making it a function of the whole commodity price schedule q.

The reason behind the form of q^s is three-folds. Firstly, instead of focusing on consumer's attention on commodity tax t, we set q^s to be the function of q to include a wider range of psychological underpinning of misperception on cost of purchasing one unit of good. Both Chetty, Looney and Kroft (2009) and Taubinsky and Rees-Jones (2017) have proved the existence of misperception of commodity tax in the United States. Still many other factors affect people's attention on prices like left-digit bias and salience of accompanying expense not included in the price tag. For example, left-digit bias is quite common in car market (Busse et al. 2013; Lacetera, Pope and Sydnor 2012), which means q_j^s depends on the absolute level of q_j. Allcott and Wozny (2014) find out that consumers undervalue future costs of gasoline when they purchase automobiles. What's more, through natural experiments in online auctions, Brown, Hossain and Morgan (2010) reveal that disclosure of shipping charges could affect consumer's demands. Hidden shipping charges indicates consumers' perceived price is lower than the actual price. The scale of number itself could affect people's perception. Roger, Roger and Schatt (2018) points out that even the financial analysts process small prices and large prices differently. Therefore, making q^s a function of q rather than t better accommodates the above physiological bias. Secondly, although it may not be intuitive to see how q_k affects q_j when $k \neq j$, the theoretical support of this relationship is strong. The theory of endogenous attention well explains this point. For example, Gabaix (2014) finds out that attention on price of one good increases with consumer's expenditure share on that good. Since the amount of good consumer purchases depends on the whole price vector in his choice set, it is not surprising to have q_j^s influenced by price vector q for any j. Similar logic could be found in portfolio choice. Mondria (2010) models the attention allocation of portfolio investors and demonstrates that rationally inattentive investors tend to observe a linear combination of two uncorrelated asset payoffs so that if there is good news about one asset they would attribute part of the effect to the other asset although the two assets are actually uncorrelated. In the last, we collect some empirical evidence in the literature on relationships between q_k and q_j. The first one relates to left-digit bias. Busse et al. (2013) finds out that buyers display higher left-digit bias for less expensive vehicles. The second one concerns with nominal illusion. Investors are likely to think in nominal dollar terms rather than percentage changes when there might be a change in stock prices (Shue and Townsend 2019). The third is about attention in portfolio choice. Investors often use information about some assets to value other assets (Hameed et al. 2015). Even professional investors have information processing constraint and the increase in attention on certain assets could generate an increase in perceived volatility of the other assets delegated to the same specialist (Corwin and Coughenour 2008).

To simplify calculation, we assume away heterogeneity among individuals in observed prices. The solution to the sparse-max problem is a set of conditional demand $c_n^s(q, y_n, z_n/n), x_{in}^s(q, y_n, z_n/n)$. The indirect utility function generated from sparse-max is $v_n^s\left(q, y_n, z_n/n\right) \equiv u\left(c_n^s, x_n^s, z_n/n\right)$.

We learn from Gabaix (2014) that classical propositions may not be robust in sparse model. So we turn to find as-if rational consumers who make same consumption choices with inattentive consumers. The following rational maximization problem describes the decisions of an as-if rational consumer with ability n:

$$\max_{c_n,x_n} u\left(c_n, x_n, z_n/n\right), s.t. c_n + \sum_i q_i^s x_{in} = \bar{y}_n\left(q, q^s, y_n, z_n/n\right). \tag{2}$$

The first-order condition is

$$u_{x_i}/u_c = q_i^s, \tag{3}$$

which is exactly the determination condition of marginal substitution rate in a sparse-max problem. $\bar{y}_n$ in the budget constraint ensures the scale of consumption bundle be the same as actual choice of an inattentive consumer. From the as-if rational maximization problem we get conditional demand function as $x_{in}^r(q^s, \bar{y}_n, z_n/n)$, $c_n^r(q^s, \bar{y}_n, z_n/n)$, and an indirect utility function $v_n^r\left(q^s, \bar{y}_n, z_n/n\right) \equiv u\left(c_n^r, x_n^r, z_n/n\right)$.

Then we feel free to use envelop theorem in this as-if rational maximization problem and other propositions in traditional model. To link inattentive behavior with rational choices, we specify the properties of virtual disposable income $\bar{y}_n$ as

Lemma 1. *The virtual disposable income $\bar{y}_n$ has the following properties:*

$$\frac{\partial \bar{y}_n}{\partial z_n} = \sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^s}{\partial z_n}; \frac{\partial \bar{y}_n}{\partial y_n} = 1 + \sum_i \left(q_i^s - q_i\right) \frac{\partial x_{in}^s}{\partial y_n}.$$

The first equation indicates that unlike exogenously given disposable income y_n in stage 2, virtual income $\bar{y}_n$ is influenced by labor income z_n. A larger gap between perceived price and actual price tends to increase influence of z_n on $\bar{y}_n$. $\frac{\partial x_{in}^s}{\partial z_n}$ could be related to conditional commodity demand elasticities with respect to labor supply. A higher $\frac{\partial x_{in}^s}{\partial z_n}$ contributes more to influence of z_n on $\bar{y}_n$. The second equation shows the difference between y_n and $\bar{y}_n$. A larger gap between q_i^s and q_i and a higher income effect on demand of commodity i both enlarge that difference. When there is no misperception on commodity price, $\bar{y}_n = y_n$ so that $\frac{\partial \bar{y}_n}{\partial z_n} = 0$ and $\frac{\partial \bar{y}_n}{\partial y_n} = 1$. We could use lemma 1 and $v_n^s = v_n^r$ to express $\frac{\partial v_n^s}{\partial z_n}$ and $\frac{\partial v_n^s}{\partial y_n}$ as

$$\frac{\partial v_n^s}{\partial z_n} = \frac{\partial v_n^s}{\partial z_n} + \frac{\partial v_n^s}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial z_n}; \frac{\partial v_n^s}{\partial y_n} = \frac{\partial v_n^c}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial y_n}. \tag{4}$$

Equations in (4) connects properties of indirect utility functions of behavioral agents and traditional agents. Similarly, we then derive the connections between expenditure functions of the two kinds of agents. Consider the conditional expenditure-minimizing problem which is dual to the as-if rational utility-maximization problem:

$$\min_{c_n, x_n} \left(c_n + \sum_i q_i^s x_{in} \right), s.t. u\left(c_n, x_n, z_n/n\right) \geq v_n^r. \tag{5}$$

This generates compensated conditional demand function $x_n^{r*}\left(q^s, v_n^r, z_n/n\right), c_n^{r*}\left(q^s, v_n^r, z_n/n\right)$ and expenditure function $e_n^r\left(q^s, v_n^r, z_n/n\right) \equiv c_n^{r*} + \sum_i q_i^s x_{in}^{r*}$. A superscript $*$ is used to denote that demands are generated by a expenditure-minimizing problem. Envelop theorem applies for this conditional expenditure-minimizing problem of as-if rational consumers.

Dual analysis links expenditure-minimizing problem and utility-maximization problem of as-if rational consumers as $e_n^r\left(q^s, v_n^r, z_n/n\right) = \bar{y}_n\left(q, q^s, y_n, z_n/n\right)$;$v_n^r\left(q^s, \bar{y}_n, z_n/n\right) = v_n^r\left(q^s, e_n^r\left(q^s, v_n^r, z_n/n\right), z_n/n\right)$. Notice that the expenditure e_n^r does not equal to real expenditure of a behavioral agent since they face different actual prices. Define real expenditure e_n^s as

$$e_n^s\left(q, v_n^s, z_n/n\right) = c_n^{r*}\left(q^s, v_n^s, z_n/n\right) + \sum_i q_i x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right). \tag{6}$$

The properties of e_n^s is characterized in the following lemma:

Lemma 2. *The real expenditure function e_n^s has the following properties: Take partial derivatives of e_n^s on v_n^s and z_n separately, we have*

$$\frac{\partial e_n^s}{\partial v_n^s} = \left(\frac{\partial v_n^s}{\partial y_n}\right)^{-1}; \frac{\partial e_n^s}{\partial z_n} = Q_n^s; \frac{\partial e_n^s}{\partial q_j} = \sum_i \left(q_i - q_i^s\right) \sum_k \frac{\partial x_{in}^{r*}}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + x_{jn}^{r*}.$$

The first equation arises from the fact that $e_n^s = y_n$ for any bounded rational agents. The second equation indicates that one dollar increase in labor income leads to an Q_n^s dollars increase of real expenditure. The third equation implies a failure of symmetry of the Slutksy matrix due to misperception on commodity prices. Based on lemma 1 and lemma 2, modified Roy's identity and Slutsky equation in stage 2 maximization could be defined.

Lemma 3. *The modified Roy's identity expressed in behavioral demand is*

$$\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n} = -x_{jn}^s + \sum_i (q_i^s - q_i) \sum_k \frac{\partial q_k^s}{\partial q_j} \frac{\partial x_{in}^{r*}}{\partial q_k^s}. \tag{7}$$

The modified Slutsky equation expressed in behavioral demand is

$$\frac{\partial x_{in}^s}{\partial q_j} = \sum_k \frac{\partial x_{in}^{r*}(q^s, v_n^r, z_n/n)}{\partial q_i^s} \frac{\partial q_k^s}{\partial q_j} + \frac{\partial x_{in}^s}{\partial y_n} \left(\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n} \right). \tag{8}$$

The left hand side of modified Roy's identity is impact on indirect utility of change in q_j measured in disposable income. The second item on the right hand side is new compared with rational agent models. While the first item on the right hand side could be interpreted as mechanism effect due to increase in q_j, the second item should be explained as behavioral effect. In traditional model with rational agents, the second item equals zero since first-order condition on x requires that at agent's optimal choice, marginal increase in utility caused by an additional unit of consumption to be offset by marginal penalty on utility due to tightening of budget constraint. However, misperception would interrupt such equality as marginal increase in utility caused by an additional unit of bounded rational agent's consumption will be offset by marginal penalty on utility due to tightening of corresponding as-if rational agent's budget constraint. In other words, the second item arises because we cannot directly apply envelop theorem to a sparse-max problem.

The first item on the right hand side of the Slutsky equation is expressed with as-if rational agent's Hicksian demand and takes into consideration the possible influence of q_j on perceived price of each non-numerate commodity. The second item replace x_n^s in rational agent's model with $\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n}$ due to behavioral effect in modified Roy's identity alters expression of the influence on indirect utility caused by a change in q_j. Both (7) and (8) could be seen as applications of general form of Roy's identity and Slutsky equation of inattentive consumers in Farhi and Gabaix (2020) using misperception as a specific kind of inattention.

B. Individual Optimization: Stage 1

In the first stage, behavioral consumer with skill n chooses labor income z_n and disposable income y_n faced with perceived labor tax schedule $T^s(z_n)$ to maximize conditional indirect utility determined in stage 2. In this sparse-max problem, consumer n believes that the relationship between his indirect utility and the choice variables in stage 1 takes the form of behavioral indirect utility function $v_n^s(q, y_n, z_n/n)$. The trade-off between z_n and y_n is perceived to be $y_n = z_n - T^s(z_n)$ so that perceived price of z_n is $-Q_n^s(q, Q_n, \mathbf{Q})$. Here we assume perceived marginal retention rate of individual n depends on his real marginal retention rate Q_n, marginal retention rate of other people (of different income level) Q_m and commodity tax schedule q. This setting is not only supported by endogenous allocation of attention model in Gabaix (2014), but also has rich empirical background. Misperception on income tax rate is documented in Brown (1969), Fujii and Hawley (1988) and Romich and Weisner (2000). Taxpayers not only find it hard to give their correct marginal tax rate in surveys, but also mistaken a lump sum tax reform as marginal tax reform (Feldman, KatuščÁk and Kawano 2016). Some research explore the factors that determine such misperception. The first strand shows how misperception is influenced by other agents' marginal tax rate. de Bartolome (1995) uses a laboratory experiment to find that taxpayers tend to behave as if their marginal tax rate is given by their average tax rate. Perceiving and responding to the average price instead of actual price is defined as ironing heuristic in Liebman and Zeckhauser (2004). Rees-Jones and Taubinsky (2019) find out that 43% of the population irons. Bénabou and Tirole (2006) and Alesina, Stantcheva and Teso (2018) propose and verify that people are overconfident of achieving high incomes. Then it could be reasonably deduced that people use marginal tax rates of high incomes to shape their actual tax rates. Prices of commodities could also be determinant of Q_n^s. For instance, as education or financial literacy education helps consumers to be more rational, the cost of education affects people's willingness in taking education and therefore shapes misperception. Money illusion could also confuse taxpayers on how much tax they have paid in real price, thus inflation (price of money) may influence perception of income tax rate as in Katseli-Papaefstratiou (1979).

Based on these assumptions, the stage-1 maximization problem could be written as

$$\text{smax}_{z_n, y_n | Q_n^s} v_n^s \left(q, y_n, z_n/n\right), s.t. y_n = z_n - T\left(z_n\right).$$

Spares-max optimization requires the marginal rate of substitution between z_n and y_n equal to perceived relative price. Thus we have

$$\frac{\partial v_n^s}{\partial z_n} / \frac{\partial v_n^s}{\partial y_n} = -Q_n^s. \tag{9}$$

Similar to the analysis in stage 2, we also use as-if rational agents to explore properties of unconditional indirect utility function V_n^s. First we reconstruct the stage 1 sparse-max problem to be

$$V_n^s(q, Q_n, \mathbf{Q}, R_n) = \mathrm{smax}_{y_n, z_n} v_n^s(q, y_n, z_n); s.t. y_n = Q_n z_n + R_n,$$

in which $R_n \equiv z_n T'(z_n) - T(z_n)$ is generalized revenue when we view z_n as a kind of good. We use the expression "generalized" rather than "virtual" in order to distinguish R_n from the revenue of an as-if rational agent. $Q_n = 1 - T'(z_n)$ is real marginal retention rate. The labor income function is therefore $z_n(q, Q_n, \mathbf{Q}, R_n)$. By imposing $T^s(z_n)$ on a rational agent, we get the following as-if rational maximization problem generating same labor income:

$$V_n^r(q, Q_n^s, \bar{R}_n) = \max_{y_n, z_n} v_n^s(q, y_n, z_n); s.t. y_n = Q_n^s z_n + \bar{R}_n(q, Q_n, Q_n^s, R_n).$$

$\bar{R}_n$ is virtual generalized revenue which satisfies $\bar{R}_n(q, Q_n, Q_n^s, R_n) = R_n - (Q_n^s - Q_n) z_n$. Perceived marginal retention rate Q_n^s satisfies $Q_n^s(q, Q_n, \mathbf{Q}) = 1 - \frac{dT^s(z_n)}{dz_n}$. Labor income function in this situation is $z_n^r(q, Q_n^s, \bar{R}_n)$. To relate the above two maximization problem, we have $z_n(q, Q_n, \mathbf{Q}, R_n) = z_n^r(q, Q_n^s, \bar{R}_n)$ and $V_n^s(q, Q_n, \mathbf{Q}, R_n) = V_n^r(q, Q_n^s, \bar{R}_n(q, Q_n, Q_n^s, R_n))$. We could also express influence of Q_n on z_n^s with the help of as-if rational agent's behavior.

Lemma 4. *The influence of Q_n on z_n^s could be decomposed as follows:*

$$\frac{\mathrm{d}z_n}{\mathrm{d}Q_n} = \left(\frac{\partial z_n^r}{\partial Q_n^s} - z_n \frac{\partial z_n^r}{\partial \bar{R}_n} \right) \frac{\partial \bar{R}_n}{\partial R_n} \frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} + z_n \frac{\partial z_n}{\partial R_n}.$$

The contents in the brackets on the right hand side is Hicksian demand of z_n^r if we take z_n^r as a special kind of good with price $-Q_n^s$ in an as-if rational agent's maximization problem. As misperception on marginal income tax rate results in discrepancy between R_n and $\bar{R}_n$, we need to multiply such Hicksian demand with $\frac{\partial \bar{R}_n}{\partial R_n} \frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}$. The second item on the right hand side could be related to income effects of a unit increase in Q_n.

C. Elasticities

In order to express optimal tax rules with measurable behavioral elasticities, we need to define elasticities under misperception at first. As is seen in lemma 3 and lemma 4, Slutsky equation and influence of Q_n on z_n^s are different from those in a traditional model. Misperception effects should be considered when specifying tax elasticities.

We follow Jacquet, Lehmann and Van der Linden (2013) and Jacobs and Boadway (2014) to define elasticities of labor income by considering the following tax reform $\tilde{T}(z,\phi) = T(z) + \phi\tau(z)$, in which $\tau(z)$ is a random policy change function and ϕ is policy change parameter. Actual marginal retention rate after the tax reform is $\tilde{Q} = 1 - T'(z) - \phi\tau'(z)$. We slightly depart from these two pieces of work as our tax reform is not restricted to a compensated (τ, ρ) form.

Define the shift function as

$$L(z_n, \mathbf{z}, n, \phi) \equiv n\tilde{Q}_n^s(q, \tilde{Q}_n, \tilde{\mathbf{Q}}) v_y^s(q, \tilde{y}_n, z_n/n) + v_l^s(q, \tilde{y}_n, z_n/n),$$

in which $\tilde{Q}_n^s$ is the perceived marginal retention rate after tax reform and $\tilde{y}$ is actual disposable income after the reform. The shift function captures the shift in first-order condition for labor income when one of the variables $z_n, \mathbf{z}, n, \phi$ changes. Since implicit function theorem gives $\frac{\mathrm{d}z_n}{\mathrm{d}\phi}|_{\phi=0} = -\frac{\partial L(z_n,z,n,0)/\partial\phi}{\partial L(z_n,z,n,0)/\partial z_n}$, we get overall impact of the tax reform on consumer's labor income as

$$\begin{aligned}\frac{\mathrm{d}z_n}{\mathrm{d}\phi} =& D_n^{-1}\tau'(z_n)\, v_y^s\left(\frac{\partial Q_n^s}{\partial Q_n} + \frac{\partial Q_n^s}{\partial \mathbf{Q}}\cdot\delta_n\right) + D_n^{-1}\tau(z_n)\left(Q_n^s v_{yy}^s + v_{zy}^s\right) \\ &+ D_n^{-1} v_y^s \int_N (1-\delta_n)\left[\tau'(z_m)\frac{\partial Q_n^s}{\partial \mathbf{Q}} - Q'(z_m)\frac{\partial Q_n^s}{\partial \mathbf{Q}}\frac{\mathrm{d}z_m}{\mathrm{d}\phi}\right]\mathrm{d}m,\end{aligned} \tag{10}$$

in which D_n is defined as

$$D_n \equiv v_y^s\left(\frac{\partial Q_n^s}{\partial Q_n} + \frac{\partial Q_n^s}{\partial \mathbf{Q}}\cdot\delta_n\right) Q'(z_n) + v_{yy}^s Q_n^s Q_n + Q_n^s v_{yz} + Q_n v_{yz} + v_{zz}. \tag{11}$$

δ_n is a Dirac distribution at point n. The impact of a tax reform could be decomposed into three parts: (1) Income effect, which captures the change in disposable income attributed to $\tau(z)$. (2) Price effect, which corresponds to the case when a reform only changes marginal income tax rate at z_n and has $\tau(z_n) = 0$ and $\tau'(z_m)|_{m\neq n} = 0$. (3) Behavioral effect. This is new compared with rational agent model, and is caused by our assumption that misperception of Q_n is a function of the whole tax schedule $\mathbf{Q}$. According to such decomposition, we could define compensated tax elasticity of labor income ε^*_{zQ}, compensated tax elasticity when marginal tax rate at z_m changes ε^*_{zQm} and income elasticity of labor income ε^I_z as

$$\varepsilon^*_{zQ} = \frac{Q_n}{z_n} D_n^{-1} v^s_y \left(\frac{\partial Q^s_n}{\partial Q_n} + \frac{\partial Q^s_n}{\partial \mathbf{Q}} \cdot \delta_n \right); \tag{12}$$

$$\varepsilon^*_{zQm} = \frac{Q_n}{z_n} D_n^{-1} v^s_y \frac{\partial Q^s_n}{\partial \mathbf{Q}} \cdot \delta_m; \tag{13}$$

$$\varepsilon^I_z = Q_n D_n^{-1} (Q^s_n v^s_{yy} + v^s_{zy}). \tag{14}$$

The expression of $\frac{dz_n}{d\phi}$ could then be transformed with these elasticities into

$$\frac{dz_n}{d\phi} = \frac{z_n}{Q_n} \left[\tau'(z_n)\, \varepsilon^*_{zQ} + \tau(z_n)\, \varepsilon^I_z / z_n + \int_N \varepsilon^*_{zQm} \left(\tau'(z_m) - Q'(z_m) \frac{dm}{d\phi} \right) (1 - \delta_n) \mathrm{d}m \right] \tag{15}$$

Use similar logic, we define uncompensated elasticity of earnings supply ε_{zn} as

$$\varepsilon_{zn} = \frac{1}{z_n} D_n^{-1} \left[\left(v^s_{zz} + v^s_{zy} Q^s_n \right) z_n + v^s_z \right]. \tag{16}$$

We also define $\varepsilon^{Qs}_{Qn} \equiv \frac{Q}{Q^s_n} \frac{\partial Q^s_n}{\partial Q_n}; \varepsilon^{\tilde{Q}s}_{Qn} \equiv \frac{Q_{\tilde{n}}}{Q^s_{\tilde{n}}} \frac{\partial Q^s_{\tilde{n}}}{\partial Q_n}; \tilde{\varepsilon}^{Qs}_{Qn} \equiv \frac{Q_n}{Q^s_n} \frac{\partial Q^s_n}{\partial \mathbf{Q}} \cdot \delta_n$ to describe impacts of real marginal tax rate on perceived marginal tax rate.

III. The Government's Problem

The government chooses to maximize social welfare taking as given consumer's behavior summarized in previous section. Assume that social welfare is the sum of non-decreasing concave social utilities $\Psi(v_n)$ on n. Before solving government's problem by mechanism design, we need to determine incentive compatibility constraint as well as economy's resource constraint.

Since government cannot observe consumers' abilities directly, the tax schedule should be designed to induce individual of type n to choose the consumption and income allocation intended for him by the government instead of allocation prepared for other ability type. The incentive compatibility constraint is similar to the form in rational agent's model as

$$u(c_n^s(y_n^s, z_n/n), x_n^s(y_n^s, z_n/n), z_n/n)) \geq u(c_n^s(y_{\tilde{n}}^s, z_{\tilde{n}}/n), x_n^s(y_{\tilde{n}}^s, z_{\tilde{n}}/n), z_{\tilde{n}}/n)); \forall (n, \tilde{n}) \in \mathrm{R}_+^2,$$

except that we use perceived disposable income $y_n^s \equiv z_n - T^s(z_n)$ rather than actual disposable income y_n, because bounded rational individuals choose allocation according to their perceived allocation plan. This inequation implies

$$n = \arg\max_{\tilde{n}} u(c_n^s(y_{\tilde{n}}^s, z_{\tilde{n}}/n), x_n^s(y_{\tilde{n}}^s, z_{\tilde{n}}/n), z_{\tilde{n}}/n)). \tag{17}$$

Combine it with total derivatives of u_n with respect to n, we get the following first-order incentive compatibility constraint :

Proposition 1. *The first-order incentive compatibility constraint taken misperception into consideration is*

$$\dot{v}_n^s = -\frac{\partial v_n^s}{\partial z_n}\left(\frac{z_n}{n} + \frac{Q_n - Q_n^s}{Q_n^s}\dot{z}_n\right), \tag{18}$$

where the dot over a variable represents a total derivative on n.

When there is no misperception on marginal tax rate, (18) would collapse into first-order incentive constraint as in Mirrlees (1976). Once taken into consideration individual's misperception, the second term in the bracket on the right-hand side of (18) is new due to failure of traditional envelop theorem with inattentive individuals. It is not surprising that the new item has $\frac{Q_n - Q_n^s}{Q_n^s}$ in it since first-order incentive constraint should reflect bounded rational consumer's behavior. Intuitively, if consumer n perceives marginal tax rate to be higher than the real value, which means $Q_n > Q_n^s$, he would be more likely to mimic labor supply of individuals with lower ability.

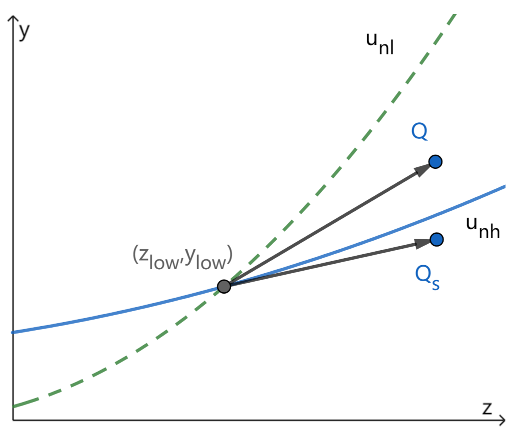
The first-order approach is valid for characterizing the second-best optimum with misperception if the following Spence–Mirrlees and monotonicity condition expressed with indirect utility function is met:

$$v_y^s \frac{\partial\left(v_z^s/v_y^s\right)}{\partial z_n}\dot{z}_n \leq 0. \tag{19}$$

Therefore, we assume that $v_y^s > 0$, $\dot{z}_n > 0$ and $\frac{\partial\left(v_z^s/v_y^s\right)}{\partial z_n} < 0$ hold in the analyses that follow to guarantee the condition implied in (19). $v_y^s > 0$ means that higher disposable income improves consumer's well-being. $\dot{z}_n > 0$ indicates that high-ability individual gains higher before-tax income. $\frac{\partial\left(v_z^s/v_y^s\right)}{\partial z_n} < 0$ means individual with lower ability has steeper indifference curve.

To be more specific, we illustrate how Q^s influence incentive constraint with agents of two different types in figure 1. Figure 1 displays the indifference curves of a high ability individual (solid line) and a low ability individual (dashed line). The two indifference curves intersect at the bundle (z_{low}, y_{low}). The indifference curve of the low-skilled workers is steeper than that of the high-skilled worker . For rational agents, as in Jacquet and Lehmann (2016), the bundle prepared for high ability individual should lie under the dashed line but over the solid line to ensure that individuals choose allocation intended for them separately. For example, the government should provide another bundle at point Q in the figure. As retention rate is defined as $\frac{y_{high}-y_{low}}{z_{high}-z_{low}}$ when there are only two bundles, the slope of vector Q corresponds to actual retention rate. However, if agents misperceive Q as Q_s and $Q > Q_s$, leading to a perceived bundle lie under the indifference curve of high ability individual, the high ability agent then has the incentive to report himself being a low-ability agent.

Figure 1: Illustration on how Q^s influence first-order incentive compatibility constraint



By contrast, misperception on commodity price q^s does not enter into incentive compatibility constraint directly. In section IV we could see how q^s modifies the impact of q on incentive compatibility constraints.

Assume that government has a fixed expenditure R. The resource constraint of the economy is

$$\int_N \left(z_n - c_n^{r*}\left(q^s, v_n^r, z_n/n\right) - \sum_i x_{in}^{r*}\left(q^s, v_n^r, z_n/n\right) - R \right) f(n)\mathrm{d}n = 0. \tag{20}$$

By Walras's law, government's budget constraint holds as long as individual's budget constraint and the whole economy's resource constraint hold.

Therefore, the government's problem could be written as

$$\max_{v^s, z, q} \int_N \Psi\left(v_n^s\right) f(n)\mathrm{d}n, \tag{21}$$

subject to (20) and (18). By defining $\kappa_n = \dot{z}_n$, we can write down the Lagrangian for this optimal control problem as:

$$\begin{aligned}
\mathcal{L} \equiv & \int_N \left[\Psi\left(v_n^s\right) + \mu \left(z_n - c_n^{r*}\left(q^s, v_n^s, z_n/n\right) - \sum x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right) - R \right)\right] f(n)\mathrm{d}n \\
& + \int_N \left[\theta_n \frac{\partial v_n^s\left(q, e_n\left(q, v_n^s, z_n/n\right), z_n/n\right)}{\partial z_n} \left(\frac{Q_n - Q_n^s}{Q_n^s}\kappa_n + \frac{z_n}{n} \right) - v_n^s \dot{\theta}_n \right] \mathrm{d}n \\
& - \int_N \left(\lambda_n \kappa_n + z_n \dot{\lambda}_n\right) \mathrm{d}n \\
& + \theta_{n_{max}} v_{n_{max}} - \theta_{n_{min}} v_{n_{min}} + \lambda_{n_{max}} z_{n_{max}} - \lambda_{n_{min}} z_{n_{min}}.
\end{aligned} \tag{22}$$

The state variables are v_n^c and κ_n while the control variables are z_n and q. μ is marginal value of government income. θ_n is co-state variable associated with first-order incentive constraint. λ_n is co-state variable associated with equation $\kappa_n = \dot{z}_n$. The variable κ is new compared with Jacobs and Boadway (2014) because $\dot{z}_n$ in first-order incentive constraint adds a state variable when forming the Lagrangian.

Combine first-order conditions of government's problem with expressions of elasticities to derive optimal commodity tax formula as well as optimal income tax formula.

IV. Optimal commodity tax

We first give optimal commodity tax formula derived from the above optimum control problem, then provide alternative expression using tax perturbation method. Corlett-Hague rule and many person Ramsey rule are linked by proving the equivalence of two results. We revisit Atkinson and Stiglitz theorem and within group uniform tax rule as application of our optimal commodity tax formula.

A. Optimal tax formula

This subsection derives two expressions of optimal commodity formula. The first is directly gained from the optimal control problem defined in section III. The second is obtained using tax perturbation approach.

Optimal commodity tax formula by mechanism design. Define misperception wedges on commodity price induced by q_j as:

$$w_{jn}^{q}\left(q, y, z_n/n\right) \equiv \sum_i \sum_k \left(q_i - q_i^s\right) \frac{\partial q_k^s}{\partial q_j} \frac{\partial x_{in}^{r*}}{\partial q_k^s}, \forall j, \tag{23}$$

which is the sum of deviation of perceived commodity price from actual one weighted by q_j's substitution effect on all general goods. w_{jn}^{q} could also be used to measure the degree of nominal illusion from the perspective of compensated commodity demand, since effect of changes in q_j could be decomposed into

$$e_{q_j} \equiv \frac{\partial e_n}{\partial q_j} = w_{jn}^{q} + x_{jn}^{s},$$

so that the sum of effects on actual expenditure caused by one dollar increase in all commodity prices is

$$\sum_j q_j e_{q_j} + \frac{\partial e_n^s}{\partial p_c} = e_n^s + \sum_j q_j w_{jn}^{q}. \tag{24}$$

Money illusion is a phenomenon that people confuse nominal with real magnitudes. With rational agents, demand function is homogeneous of degree zero in all nominal prices, indicating no money illusion (Leontief 1936). Inattention is one psychological reasons behind money illusion as consumer's demand function on nominal prices and nominal income is no longer homogeneous of degree zero (Gabaix 2014). In our model, from the perspective of compensated commodity demand and expenditure function, money illusion is reflected in the second item of (24). If consumers are fully rational, expenditure function will be homogeneous of degree zero in all nominal prices indicating the second item of (24) is zero. In this way, w_{jn}^{s} measures the contribution of misperception on q_j by consumer n on his money illusion.

Based on the above definition, proposition 2 gives optimal tax formula comparable to that of a traditional case in Jacobs and Boadway (2014).

Proposition 2. *Optimal commodity tax: for $\forall j \in \{1 : I\}$, the optimal linear commodity tax q_j at the optimal nonlinear income tax satisfies:*

$$\begin{aligned} &\int_Z \sum_i t_i \sum_k \frac{\partial q_k^s}{\partial q_j} \frac{\partial x_{in}^{r*}}{\partial q_k^s} \tilde{f}(z)\mathrm{d}z + \int_Z w_{jn}^q \tilde{f}(z)\mathrm{d}z \\ =&\int_Z \Theta_z \left[\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n} \left[\frac{1}{\varepsilon_{zn}} + \left(\frac{Q_n}{Q_n^s} - 1 \right) \right] - \frac{Q\left(z\right)}{Q^s\left(z\right)} \frac{\partial Q^s}{\partial q_j} \right] \mathrm{dz}. \end{aligned} \tag{25}$$

When there exists no misperception, we have (25) reduced to optimal commodity tax expression in Jacobs and Boadway (2014). To explain the tax formula, we modify the expression in (25) as

$$\begin{aligned} &\int_Z \sum_i t_i \sum_k \frac{\partial q_k^s}{\partial q_j} \frac{\partial x_{in}^{r*}}{\partial q_k^s} \tilde{f}(z)\mathrm{d}z + \int_Z w_{jn}^q \tilde{f}(z)\mathrm{d}z \\ =&\int_Z \Theta_z \left[\left(\frac{\partial w_{jn}^q}{\partial z_n} + \frac{\partial x_{jn}^s}{\partial z_n} \right) \frac{1}{\varepsilon_{zn}} + \left(\frac{\partial w_{jn}^q}{\partial z_n} + \frac{\partial x_{jn}^s}{\partial z_n} \right) \left(\frac{Q_n}{Q_n^s} - 1 \right) - \frac{Q_n}{Q_n^s} \frac{\partial Q^s}{\partial q_j} \right] \mathrm{dz}. \end{aligned} \tag{26}$$

This formula captures the trade-off between distortions (measured in government's income) caused by commodity tax and by labor income tax. The first item on the left-hand side is similar to expression in Jacobs and Broadway (2014) and captures decrease in commodity tax revenue due to reduction in compensated commodity demand by marginally increasing linear tax rate on good j. w_{jn}^q is misperception wedge of individual with ability n on commodity tax rate. For example, if $q_j > q_j^s$, consumers over-consume good j, pushing upward t_j. In total, the left-hand side is distortion caused by linear tax on good j.

By contrast, the right-hand side shows distortion resulted from nonlinear income tax, and commodity tax could impact such distortion by relaxing or tightening the first-order incentive constraints. The composition of the right-hand side is much more complex than in Jacobs and Broadway (2014), and we decompose it into the following channels:

- Elasticities with labor supply. We begin with the item $\frac{\partial x_{jn}^s}{\partial z_n}$. Change in demand of good j correlates with labor supply through complement and substitution effects in consumer's stage 2 decision. If $\frac{\partial x_{jn}^s}{\partial z_n} > 0$, decrease in x_{jn} aggravates the distortions of the income tax on work effort by discouraging labor supply. This effect is also emphasized in Jacobs and Broadway (2014). Our modification relative to traditional model is primarily reflected in $\frac{\partial w^q_{jn}}{\partial z_n}$, indicating misperception wedge on commodity tax rate affects labor supply and then incentive constraints. For example, if $\frac{\partial w^q_{jn}}{\partial z_n} > 0$, a decrease in wedge would enlarge distortion caused by income taxation.

- Amplification effect by income tax perception. Misperception on marginal labor income tax rate plays a role in shaping commodity tax, and it acts as a multiplier on the sum of previous effects, $\frac{\partial x^s_{jn}}{\partial z_n}$ and $\frac{\partial w^q_{jn}}{\partial z_n}$. If $Q^s_n < Q_n$, individual with ability n perceives a higher marginal tax rate than government's real policy, making it attractive to deviate from z towards a lower labor supply.

- Cross impacts of commodity price on income tax misperception. By our assumption, an increase in t_j also affects Q^s . When $\frac{\partial Q^s_n}{\partial q_j} < 0$, which means perceived marginal income tax rate positively correlates to t_j, increase in t_j induces consumers to mimic labor supply of individuals with lower ability.

The above three channels separately correspond to the three items in square bracts on the right-hand side of (26). Θ_z is the net social welfare loss due to a small increase in marginal tax rate at z_n measured in terms of income, the expression of which is presented in (36) of section V. It transforms impacts on incentive constraints into impacts on marginal utility loss.

Overall, the role of indirect taxation in our setting could be divided into bias-correction and redistribution as in Allcott, Lockwood and Taubinsky (2019). However, the mechanism behind the two roles are different. The redistributive motive in Allcott, Lockwood and Taubinsky (2019) arises from preference heterogeneity. It would be zero when variation in consumption are due purely to variation in income. However in our model, consumers' preference are heterogeneous and our redistributive motive comes from substitutability between preference for commodity goods and labor supply as well as misperception to both commodity prices and marginal income tax rate. The bias-correction term is also different due to different causes of behavioral bias. Commodity tax needs to correct misperception wedge on commodity price in our model, while sin tax in their research should correct negative externality of sin goods consumption and internality caused by mismatch between decision utility and experienced utility.

Optimal commodity tax formula by tax perturbation method. We first restate the government's optimization problem for the convenience of tax perturbation approach. Then we give alternative expression of optimal commodity tax formula and its relationship with expression in proposition 2.

To capture the impact of commodity price changes on net social welfare, express social welfare function with unconditional indirect utility function V_n^s as

$$W \equiv \int_N \Psi(V_n^s(q, Q_n, \mathbf{Q}, R_n)) f(n) \mathrm{d}n.$$

Note that $V_n^s = v_n^s$ in (21). Define total tax income as

$$B \equiv \int_N T(z_n) f(n) \mathrm{d}n + \int_N \sum_i t_i x_{in}^s(q, y_n, z_n) f(n) \mathrm{d}n.$$

We learn from (22) that marginal value of government income is μ. Therefore, we could define net social welfare function expressed in government income as $W/\mu + B$. The marginal influence of commodity tax on net social welfare is

$$\begin{aligned} \frac{\mathrm{d}W}{\mathrm{d}q_j}/\mu + \frac{\mathrm{d}B}{\mathrm{d}q_j} = & \int_N \frac{\Psi' \mathrm{d}V_n^s/\mathrm{d}R_n}{\mu} \frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/dR_n} f(n)\mathrm{d}n \\ & + \int_N x_n^j f(n)\mathrm{d}n + \int_N \sum_i t_i \frac{\mathrm{d}x_n^i}{\mathrm{d}q_j} f(n)\mathrm{d}n + \int_N T'(z_n) \frac{\mathrm{d}z_n}{\mathrm{d}q_j} f(n)\mathrm{d}n. \end{aligned} \tag{27}$$

Optimal commodity tax requires that marginal influence of commodity tax be zero. Define commodity price elasticity on labor income as

$$\varepsilon_{zq_j} = \frac{q_j}{z_n} \frac{\mathrm{d}z_n}{\mathrm{d}q_j}.$$

Armed with properties of V_n^s (derived in the appendix), we express optimal tax formula in behavioral wedges and a different set of behavioral elasticities compared with (26).

Proposition 3. *Optimal commodity tax derived by tax perturbation approach: for* $\forall j \in \{1 : I\}$, *the optimal linear commodity tax q_j at the optimal nonlinear income tax satisfies:*

$$\begin{aligned}\int_Z \left(1-g_n-\sum_i t_i \frac{\partial x^s_{in}}{\partial y_n}\right) x^s_{jn}\tilde{f}(z)\mathrm{d}z &+ \int_Z \sum_i t_i \sum_k \frac{\partial x^{r*}_{in}}{\partial q^s_i}\frac{\partial q^s_k}{\partial q_j}\tilde{f}(z)\mathrm{d}z \\ &- \int_Z \left(g_n+\sum_i t_i \frac{\partial x^s_{in}}{\partial y_n}\right) w^q_{jn}\tilde{f}(z)\mathrm{d}z \\ &= -\int_Z \left(\frac{1-Q_n}{Q_n} - g_n\tau^b_n + \sum_i \frac{t_i}{Q_n}\frac{x^s_{in}}{z_n}\varepsilon_{x_i z}\right) Q_n \frac{\mathrm{d}z_n}{\mathrm{d}q_j}\tilde{f}(z)\mathrm{d}z. \end{aligned} \tag{28}$$

While (26) reflects government's trade-off between distortions (measured in government's income) caused by commodity tax and by labor income tax, (28) compares the cost and benefit of a unit of marginal increase in t_j in current tax system measured in government income. The left hand side describes the net welfare effect through changes in commodity demand when t_j increases one unit while the right hand side measures the net welfare effect through changes in labor supply. Optimal commodity tax requires that total net welfare effect of a change in q_j be zero. Details in explanation are given as follows.

The effects on the left hand side of (28) could be decomposed into three parts:

- Firstly, one unit increase of t_j transfers x_{jn} units of revenue from individual n to the government, so that net social welfare increases by $1-g_n$ with regard to each consumer. Increase in t_j also inhibit consumer's purchasing power under current labor income. As a result, government's commodity tax revenue decreases by $\sum\limits_i t_i \frac{\partial x^s_{in}}{\partial y_n} x^s_{jn}$ through income effect on individual n. These two influences corresponds to the first item on the left hand side of (28).

- Secondly, the tax increase changes commodity demands through compensated effect and then leads to changes in government's commodity tax revenue. The overall compensated effect on the population in embodied in the second item on the left hand side.

- In the last, due to misperception on commodity price, consumers have nominal illusion in the second stage of their decisions. Such illusion affects not only individual welfare directly, but also government tax revenue. The degree of nominal illusion is measured by w^q_{jn}. To be more concrete, assume that $w^q_{jn} > 0$, which means increase in q_j induces individual n to overspend compared with full-rational consumers holding utility constant. In other words, this inattentive agent would feel as if his budget is more tightened holding labor supply constant. Consequently, he deviates from optimal decisions on commodity goods consumption. His utility is dampened by $g_n w_{jn}^q$ and commodity tax revenue shrinks by $\sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} w_{jn}^q$.

The right hand side measures the net welfare effect through changes in labor supply. The increase in t_j changes labor income by ε_{zq_j}, the impacts of which on government's tax revenue is summarized by marginal retention rate Q_n multiplies items in the brackets. We leave the detailed interpretation of items in the brackets in section V because they exactly form the left hand side of (35).

B. Connect results from mechanism design and tax perturbation method

While the equivalence of optimal income tax formula derived from mechanism design and tax perturbation method is easy to prove and has been widely applied in optimal tax literature, the connection between optimal commodity tax formula from these two approaches is rarely explored. We find that many person Ramsey rule is embodied in (28), and Corlett-Hague rule is implied in (26). Thus, by proving the the equivalence of two expressions of optimal commodity tax, we could link the two commodity tax rule together.

Many person Ramsey rule with misperception. We examine many person Ramsey rule with nonlinear income tax and misperception, and find that a bias-correction and a bias-motivated redistribution term should be added to traditional "many person Ramsey rule" in our setting.

Corollary 1. *The many-person Ramsey rule with nonlinear income tax and misperception should be modified as*

$$\begin{aligned} -\frac{1}{\overline{x_j^s}+\overline{w_j^q}}\overline{\sum_i t_i \frac{\partial x_i^{r*}}{\partial q_j}} =& 1-\overline{\gamma}-\operatorname{cov}\left(\gamma, \frac{x_{jn}^s + w_{jn}^q}{\overline{x_j^s}+\overline{w_j^q}}\right) \\ & -\frac{\overline{w_j^q}}{\overline{x_j^s}+\overline{w_j^q}} + \frac{1}{\overline{x_j^s}+\overline{w_j^q}} \overline{\frac{\Theta_n}{n\tilde{f}(z)} \frac{h^*(z_n)}{h(z_n)} \frac{Q_n}{Q_n^s} \frac{\partial Q_n^s}{\partial q_j}}. \end{aligned} \tag{29}$$

in which γ_n is marginal social utility of income and is defined by

$$\gamma_n \equiv g_n + \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} + \frac{h^*(z_n)}{f(n)\,h(z_n)} \frac{\varepsilon_{zn}\Theta_n}{n} \frac{Q_n}{Q_n^s} \left[\frac{\varepsilon_z^I}{\varepsilon_{zQ}^*} \left(\frac{\partial Q_n^s}{\partial Q_n} + \frac{\partial Q_n^s}{\partial \mathbf{Q}} \cdot \delta_n \right) + \varepsilon_{e_j z} \right], \tag{30}$$

and the "bar" indicates an integral on z.

The first line of (29) is the familiar form in traditional "many person Ramsey rule" as in Diamond (1975) with multiple goods and continuous agent's type. $\bar{\gamma}$ is average marginal social utility of income, and $1-\bar{\gamma}$ captures government's revenue raising motive. The covariance term is slighted modified as we replace traditional term x_{jn} with $x_{jn}^s + w_{jn}^q$ so that the right-hand side is smaller if people of higher marginal social utility of income consume more good j and have higher positive misperception wedge w_{jn}^q. Therefore, the first line of (29) means that at the optimum, good j is more discouraged if government's need for revenues is large and if agents with low social marginal utility of income consume relatively more of good j and show higher positive misperception wedge on price of j.

The second line of (29) reflects bias-correction motive of commodity taxation. The first item in the second line of (29) corresponds to *correcting bias term* in Farhi and Gabaix (2020). Good j is more discouraged if average misperception wedge on price of good j is relatively higher than average consumption of good j. The second item in the second line is new, which corrects the bias caused by influence of q_j on perceived marginal income tax rate. Intuitively, if price of good j contributes to a higher perception of marginal income tax rate, which means $\frac{\partial Q_n^s}{\partial q_j} > 0$, then good j should be more discouraged by commodity tax at the optimum.

Allcott, Lockwood and Taubinsky (2018) point out that if commodity taxes are not fully salient, optimal commodity taxes essentially follow "many person Ramsey rule" scaled by the degree of inattention. Our work differs from theirs because we assume that consumers misperceive commodity price when they make purchase decisions, while in their setting, commodity tax rates are salient at the time of purchasing but are misunderstood when people choose labor supply. Therefore, we have the additional bias-correcting term.Besides, the scaling factor in Allcott, Lockwood and Taubinsky (2018) measures the degree that consumers' perception on commodity price is close to actual one when making labor supply decision. But our scaling factor in covariance term reflects behavioral wedge at the time people purchase goods, and it exists even if labor supply is fixed.

Corlett-Hague rule with misperception. Jacobs and Boadway (2014) find classical Corlett–Hague rule implied in the Mirrlees framework with optimal nonlinear income taxes, which is again emphasized in (25) in our model. The spirit that high complementarity to leisure pushes commodity tax upward still works in the Mirrlees framework with optimal nonlinear income taxes and misperception on prices.

As has been stressed in Christiansen (1984), the correlation between labor supply and commodity demand is akin to but not identical to compensated elasticity concepts used by Corlett and Hague (1954). Our modification with misperception is that misperception has rescaling effect on such correlation. If higher conditional commodity demand corresponds to lower labor supply and more leisure time, commodity tax tends to be positive. A negative correlation between commodity price misperception wedge and labor supply or perceiving marginal income tax rate to be higher than actual value would enlarge such force on commodity tax. Another two concerns which are not included in Corlett–Hague rule are impact of commodity price on income tax perception and then on government's redistribution, and correction motive on commodity tax misperception.

In brief, introducing misperception into analyzing of optimal income redistribution with heterogeneous agents does not change the nature of the Corlett-Hague conclusions. Therefore, we obtain both modified many person Ramsey rule and analogue of the Corlett-Hague rule for optimal linear commodity tax rate with misperception using different analytical methods. Next, we present the link between results of two methods and between the two tax rules.

Equivalence of optimal tax expression. Using decision rules of behavioral consumers, we could transform (28) into expression in (25) or vice versa. Therefore, the two expressions for optimal commodity tax are equivalent.

Proposition 4. *Optimal commodity tax formula derived from tax perturbation method is equivalent to optimal commodity tax formula solved by mechanism design approach.*

To the best of our knowledge, proposition 4 is the first to establish equivalence between mechanism design and tax perturbation method with regard to commodity tax in a heterogeneous agents mixed taxation environment. The equivalence still exists when consumers are fully rational. Intuitively, the point to be emphasized here is the embodied link between two classical optimal commodity taxation rules. The two rules capture government's trade-off in deciding commodity tax from different aspects. The Corlett-Hague rule implies a trade-off between distortions caused by commodity tax and by labor income tax. The many person Ramsey rule reflects the trade-off between raising government's revenue and improving the social welfare.

C. Implications

Atkinson and Stiglitz theorem revisited. We then revisit the Atkinson and Stiglitz theorem when preferences are weakly separable between commodities and labor to see whether linear commodity taxes are superfluous when individual's utility is weakly separable between commodities and labor. With utility function taking the form $u(h(c_n, x_n), z_n/n)$, we find that tendency to implement uniform commodity tax is hindered by two forces presented in corollary 2.

Corollary 2. *When individual's utility is weakly separable between commodities and labor, linear commodity tax t_j still plays an efficiency role in alleviating distortion caused by incentive constraint as long as misperception on labor income tax is influenced by price of commodity j. For $\forall j \in \{1 : I\}$, the optimal t_j satisfies:*

$$\int_Z \sum_i t_i^s \sum_k \frac{\partial q_k^s}{\partial q_j} \frac{\partial x_{in}^{r*}}{\partial q_k^s} \tilde{f}(z)\mathrm{d}z = -\int_Z \Theta_z \frac{Q_n}{Q_n^s} \frac{\partial Q_n^s}{\partial q_j} \mathrm{d}z, \tag{31}$$

in which $t_i^s = q_i^s - 1$ is perceived commodity tax.

If $\frac{\partial Q_n^s}{\partial q_j} = 0, \forall n$, then it is obvious that $t_i^s = 0, \forall i$ is a sufficient condition of (31). The Atkinson and Stiglitz theorem here should be modified to require all perceived commodity taxes to be zero. Since real tax rate might not coincide with perceived tax rate, government still need non-uniform commodity tax, otherwise government should make additional efforts to eliminate such misperception to achieve a uniform tax system. When influence of q on perceived marginal income tax rate is non-zero, we find that misperception on commodity tax allows for a more progressive income tax system, indicating another channel through which commodity taxes are not superfluous in our setting and government could make use of individual's inattention for a given desire to redistribute income.

In brief, misperception introduces two forces against uniform tax schedule to achieve the second-best optimum even under weakly separable preference. The first is that misperception on q causes inefficiency among commodity demands. The second force arises because inattention makes it possible to utilize re-distributive role of q through influence of q on Q_n^s. Farhi and Gabaix (2019) also point out that uniform ad valorem commodity taxes are not optimal in general when consumer's decision utility differs from experienced utility. Our work could rather shed light on how misperception on prices departs commodity taxation from uniformity.

Within group uniform tax rule revisited. Same intuition applies when we revisit uniform taxation rule for any separable subgroup of commodities in Deaton (1979). When individuals are rational, the assumption that within-group Engel curves are linear is equivalent to the specification that sub-utility function of commodities in this group is homothetic. With inattentive consumers, the two assumptions are no longer tantamount and we cannot arrive at uniform tax from neither one. Corollary 3 describes how within group uniform tax rule is hampered by misperception.

Corollary 3. *Neither homothetic sub-utility function nor linear within-group Engel curves is sufficient for uniform taxation on x when consumers misperceive commodity price q.*

To be more concrete, we first notice that conditional compensated demand is no longer homogeneous of degree zero in all commodity price (including price of numerate good). In other words, we only have x_{in}^{r*} homogeneous of degree zero in perceived prices, which indicates

$$\sum_i \left(q_i^s - q_i\right) \frac{\partial x_{kn}^{r*}}{\partial q_i^s} + \sum_i q_i \frac{\partial x_{kn}^{r*}}{\partial q_i^s} = -\frac{\partial x_{kn}^{r*}}{\partial q_c}. \tag{32}$$

Under both preference structure specified in corollary 3, the conditional compensated demand would have the following form

$$x_{jn}^{r*} = a_j\left(q\right) + b_j\left(q\right) d_n, \forall j. \tag{33}$$

d_n is same for all j. Suppose government impose $q_i = q$ for $\forall i \in \{1 : I\}$. According to (32), we have

$$\sum_i \frac{\partial x_{kn}^{r*}}{\partial q_i^s} = \frac{1}{q}\left[-\frac{\partial x_{kn}^{r*}}{\partial q_c} - \sum_i \left(q_i^s - q\right)\frac{\partial x_{kn}^{r*}}{\partial q_i^s}\right], \forall k \in \{1 : I\}.$$

Combine it with (25), we find that optimal t should satisfy

$$\begin{aligned} -b_j \frac{t}{q} \overline{\frac{\partial d_n}{\partial q_c}} \frac{\partial q_j^s}{\partial q_j} - \frac{t}{q} \overline{\frac{\partial d_n}{\partial q_c}} \sum_{k \neq j} b_k \frac{\partial q_k^s}{\partial q_j} - \frac{1}{q}\overline{w_j^q} \\ = b_j \overline{\frac{\Theta_n}{f\left(n\right)} \frac{\partial d_n}{\partial z_n}\left[\frac{1}{\varepsilon_{zn}} + \left(\frac{Q_n}{Q_n^s} - 1\right)\right]} \\ + \overline{\frac{\Theta_n}{f\left(n\right)} \frac{\partial w_j^q}{\partial z_n}\left[\frac{1}{\varepsilon_{zn}} + \left(\frac{Q_n}{Q_n^s} - 1\right)\right]} - \overline{\frac{\Theta_n}{f\left(n\right)} \frac{Q_n}{Q_n^s} \frac{\partial Q_n^s}{\partial q_j}}. \end{aligned} \tag{34}$$

The "bar" means an integral on density of z. When consumers are fully rational, b_j on both sides cancels out, implying optimal commodity tax is uniform. However, misperception induces deviation from uniform tax obviously. The force behind such deviation is three-folds: The first is commodity price misperception wedge w_j^q in the third item on the left hand side of (34) as well as in the second item on the right hand side. Secondly, with misperception, actual price of commodity j might impact individual's perception of other commodities' prices. The third is the influence of q_j on Q_n^s in the third item on the right hand side of (34). The summation of all these forces hardly equals to zero, nor could we extract b_j from that, thus solution of t from different j (34) generally varies with j.

To gain more intuition, recall that in the traditional case, when the sub-utility of certain goods is homothetic and weakly separable from the numerate good, the goods within the sub-utility function could be aggregated into a composite commodity. This is the reason why there should be no commodity-tax differentiation within that group. However, with misperception on prices, such composite commodity no longer exists and the illustration using current setting comes as follows:

Before the proof by contradiction, we get consumer n's decision rule in his stage-2 maximization problem as $\frac{u_h h_j}{u_c} = q_j^s, \forall j$ and $y_n - c_n^s = \sum_i q_i x_{in}^s$. Assume there exists a composite good h which generates utility h with price $\bar{q}$. Then individual n chooses c_n and h in his second stage decision. The maximization problem writes

$$\max_{c_n, h_n} u\left(c_n, h_n, z_n/n\right), s.t. c_n + \bar{q} h_n = y_n.$$

The first-order condition of the problem is $\frac{u_h}{u_c} = \bar{q}$. From the two budget constraint we get expression of $\bar{q}$ as

$$\bar{q} = \frac{\sum_i q_i x_{in}^s}{h\left(x_{1n}, x_{2n}, ...x_{In}\right)}.$$

In the traditional case with rational agents, $\bar{q}$ is irrelevant to subgroup consumption expenditure $y_n - c_n$ and should be homogeneous of degree one with regard to q, which means one percent increase in all q_j leads to increase in $\bar{q}$ of the same degree. By contrast, with commodity price misperception, such property does not hold generally. According to expression of $\bar{q}$ we have

$$\sum_j \frac{\partial \bar{q}}{\partial q_j} q_j = \bar{q} + \frac{\bar{q}}{y_n - c_n} \sum_j \sum_i \left(q_i - q_i^s\right) q_j \frac{\partial x_{in}^s}{\partial q_j}.$$

The second item on the right hand side hinders $\bar{q}$ to be homogeneous of degree one. Therefore, goods within subgroup utility function h can not project to a composite good so that uniform subgroup commodity tax no longer applies with misperception on prices.

Even if $q_j^s = q_j$, we cannot immediately draw the conclusion of uniform taxation. Another difference in our analysis with the traditional case of rational agents is that commodity price could affect people's perception of marginal income tax rate. As a result, government has the incentive to make use of differentiated commodity price to redistribute income. This reason is similar to explanation for corollary 2 about superfluousness of commodity tax.

In summary, misperception in our model on the one hand excludes the existence of a composite commodity, on the other hand gives government the chance to meet its distributive motive for a step further, thus precludes uniform taxation on x. This result could be generalized to the case where utility function consists of more than one sub-utility function, each of which is homothetic and weakly separable from numerate good. Within group uniform commodity taxation generally does not apply for the same reason.

V. Optimal labor income tax

In this section, we first provide optimal income tax formula under misperception, and compare our results with Farhi and Gabaix (2020) as well as Jacobs and Boadway (2014). Then we examine optimal income tax rate at endpoints of the skill distribution, and find that misperception departs optimal marginal tax rate from zero at the endpoints.

A. Optimal income tax formula

Define misperception wedge of labor income tax as $\tau_n^b \equiv \frac{Q_n^s - Q_n}{Q_n}$, which is the degree of deviation of perceived marginal retention rate from actual one. Use $g_n \equiv \frac{\Psi'(v_n^s)v_y^s}{\mu}$ to denote social welfare weight on individual n. To facilitate comparison with the earlier work of Saez (2001) and Farhi and Gabaix (2020), denote the cumulative distribution of earnings by $\tilde{F}(z_n)$ and use $F(n) \equiv \tilde{F}(z_n)$ to get optimal income tax formula.

Proposition 5. *Optimal income tax: the nonlinear income tax $T(z)$ satisfies the following expres-*

sion at all points of differentiability:

$$\frac{T'(z)}{1-T'(z)} + \sum_i \frac{t_i}{Q}\frac{x^s_{in}}{z}\varepsilon_{x_i z} - g_n\tau^b_n = -\frac{1}{\varepsilon^*_{zQ}/\left(\varepsilon^{Qs}_{Qn} + \tilde{\varepsilon}^{Qs}_{Qn}\right)}\frac{1}{z\tilde{f}(z)}\Theta_z$$
$$+ \frac{T''(z)}{1-T'(z)}\frac{1}{z\tilde{f}(z)}\frac{n}{\varepsilon_{zn}}\int_Z\left(\frac{\varepsilon^*_{mQ_n}}{\varepsilon^*_{mQ_m}/\left(\varepsilon^{Qms}_{Qm} + \tilde{\varepsilon}^{Qms}_{Qm}\right)}\Theta_m(1-\delta_z)\right)\mathrm{d}m \tag{35}$$

with

$$\varepsilon_{x_i z} = \frac{z_n}{x^s_{in}}\left(\frac{\partial x^s_{in}}{\partial y_n}Q_n + \frac{\partial x^s_{in}}{\partial z_n}\right)$$

and

$$\Theta_z \equiv \theta_n v^s_y/\mu = \int_z^{z_{\max}} e^{-\int_z^{z'}\rho(s)\mathrm{d}s}\left(1 - g - \sum_i t_i\frac{\partial x^s_i}{\partial y}\right)\tilde{f}(z')\mathrm{d}z'; \rho = -\frac{\varepsilon^I_z}{\varepsilon^*_{zQ}/\left(\varepsilon^{Qs}_{Qz} + \tilde{\varepsilon}^{Qs}_{Qz}\right)}\frac{1}{z} \tag{36}$$

Overall, (35) could be seen as a modified version of the ABC-formula of Diamond (1998). It is a combination of result in Jacobs and Boadway (2014), which explore optimal mixed taxation with fully rational individuals, and result in Farhi and Gabaix (2020), which derive optimal income tax formula with inattentive agents, but we define substitution elasticities more precisely than Jacobs and Boadway (2014) [1], and use mechanism design rather than tax perturbation method in Farhi and Gabaix (2020). A detailed explanation on formation of (35) is provided in below.

The first and second items on the left-hand side of optimal income tax formula captures impacts on government's tax revenue when labor income z increases one unit. Tax revenue increases not only due to direct marginal tax on labor earnings $T'(z)$ but also due to indirect commodity taxes since income effect on x brings about additional tax burden on labor supply. These two effects are just as in Jacobs and Broadway (2014). The third item on the left-hand side implies a correction on misperception of labor income tax of inattentive consumers who earn labor income z. Since individuals earning z misperceive income tax as $T^s(z)$, an increase in real marginal tax rate could induce an additional perceived decrease in disposable income as $Q^s(z) - Q(z)$, indicating a direct utility cost $g(Q^s(z) - Q(z))$. For example, $\mathrm{d}T^s/\mathrm{d}z < \mathrm{d}T/\mathrm{d}z$, which means $Q^s(z) > Q(z)$,pushes toward a higher tax rate.

[1] Jacobs and Boadway (2014) mis-defined $\varepsilon_{x_i z}$ to be conditional labor elasticity of commodity demand and omitted income effect on conditional commodity demand.

The first item on the right-hand side shows a slight modification of expression in Saez (2001).Note that $\varepsilon^*_{zQ}/\left(\varepsilon^{Qs}_{Qn}+\tilde{\varepsilon}^{Qs}_{Qn}\right)$ replaces compensated tax elasticity of labor income ε^*_{zQ}. This is because government needs to correct the price effect of misperception wedge on marginal tax rate. The module Θ_z is familiar and captures the net social welfare loss due to a small increase in marginal tax rate at z measured in terms of income. To interpret the structure of Θ_z, note that an increase of marginal tax rate at z has the following impacts:

- It leads to changes of marginal tax rate at income levels above z, so government get additional labor income tax revenue from these individuals, which corresponds to 1 in the brackets of (36).
- The g in (36) implies that such a change causes a direct utility loss on individuals through reduction in disposable income.
- Since reduction in disposable income changes demand of commodities, indirect effects on government's revenue are embodied in $\sum_i t_i \frac{\partial x_i^s}{\partial y}$.

It also worth noting that an increase in total income tax has an income effect on labor supply and the tax schedule is non-linear so that compensated effect of tax should also be taken into consideration. This explains the existence of exponential term outside the brackets.

Remember $Q(z)$ has externalities on other people's income tax perception since we have assumed that $Q^s(m)$ is affected by $Q(z)$ when $m \neq z$. Hence optimal income tax should also correct such effects, which explains the second item on the right-hand of optimal tax formula. $1-\delta_n$ arises in the the second item because we need to deduct the overlaps of effects described by the previous two items. The same expression in (35) could be recovered using the widespread "tax perturbation" method. See appendix for details.

B. Implication

Optimal income tax rate at endpoints of the skill distribution. From first-order conditions on $v_{n_{max}}$ and $v_{n_{min}}$ of government's problem, we have $\theta_{n_{max}} = \theta_{n_{min}} = 0$. Therefore, marginal income tax rates at the endpoints of the skill distribution satisfy:

$$T'(z_n) = g_n\left(T'(z_n) - T^{s\prime}(z_n)\right) + \frac{T''(z_n)}{f(n)} \int_N \left(\Theta_{\tilde{n}} \frac{z_{\tilde{n}}}{\tilde{n}} \frac{Q_{\tilde{n}}}{Q^s_{\tilde{n}}} \frac{\partial Q^s_{\tilde{n}}}{\partial Q_n} \varepsilon_{\tilde{z}\tilde{n}}\right) \mathrm{d}\tilde{n} - \sum_i t_i \frac{x^s_{in}}{z_n} \varepsilon_{x_i z}, n \in \{n_{\min}, n_{\max}\}. \tag{37}$$

Compared to rational agent's model in Sadka (1976) and Seade (1977) where $T' = 0$ at the endpoints of the skill distribution, misperception in our model departs optimal marginal tax rate from zero at the endpoints. The first and second item on the right hand side of equation are throughly discussed in Farhi and Gabaix (2020) using specific income tax perception function which captures both schmeduling in Liebman and Zeckhauser (2004) and overconfidence of achieving high incomes in Bénabou and Tirole (2006) and Alesina, Stantcheva and Teso (2018). For top incomes, their findings on relationships between types of misperception and top income marginal tax rates could also be directly seen from (37). To be concrete, Farhi and Gabaix (2020) assume that agents are only influenced by incomes higher than theirs to describe overconfidence, corresponding to $\frac{\partial Q^s_{\tilde{n}}}{\partial Q_n} = 0, \forall n < \tilde{n}$ in our model, while in the schmeduling case they assume that one's average tax rate affects his perception, implying $\frac{\partial Q^s_{\tilde{n}}}{\partial Q_n} > 0, \forall n, \tilde{n}$. Therefore, being overinfluenced by higher incomes (overconfidence) pushes the second item on the right hand side of (37) upward relative to being overinfluenced by lower incomes (schmeduling), leading to a higher marginal tax rate for top incomes. As for difference with their work, the third item on the right hand side of (37) is additional and reflects indirect effect of misperception of commodity price on income tax design.

VI. A simple specification with two goods

In this section, we use a concrete model with misperception of commodity price and marginal tax rate to simplify the discussion and act as preliminary steps for application of our theory with specific goods and psychological issue.

We make several simplifying assumptions: firstly, there is only one kind of general goods x with actual price q, which is perceived as q^s. Consumer's utility function shares similar form as in Saez (2002) and Golosov et al. (2013), which is separable between consumption and labor. The form of consumer n's utility is:

$$u\left(c_n^s, x_n^s, z_n/n\right) = \beta \ln c_n^s + (1-\beta)\ln x_n^s - \frac{1}{\sigma}\left(\frac{z_n}{n}\right)^{\sigma}. \tag{38}$$

It is then easy to find that optimal commodity tax satisfies

$$\frac{q^s-1}{q^s}\frac{\partial q^s}{\partial q}\left(\frac{1}{1-\beta}q^s+\frac{1}{\beta}q\right)^{-1} = \frac{\int_Z \Theta_z \frac{Q}{Q^s}\frac{\partial Q^s}{\partial q}\mathrm{d}z}{\int_Z \left(z-T\left(z\right)\right)\tilde{f}(z)\mathrm{d}z}, \tag{39}$$

and optimal income tax schedule satisfies

$$\begin{aligned}\frac{T'(z)}{1-T'(z)} - g_n\tau_n^b = &-\frac{D\left(z\right)}{\tilde{f}(z)}\frac{z-T\left(z\right)}{Q^s\left(z\right)}\Theta_z \\ &-\frac{D\left(z\right)}{\tilde{f}(z)}\frac{zT''(z)}{1-T'(z)}\frac{n}{\sigma}\left(\frac{z}{n}\right)^{-\sigma}\int_Z \Theta_m \frac{Q\left(m\right)}{Q^s\left(m\right)}\frac{\partial Q^s\left(m\right)}{\partial Q\left(z\right)}\left(1-\delta_z\right)\mathrm{d}m \\ &-\frac{\left(q-1\right)q^s}{\left(q^s-1\right)\frac{\partial q^s}{\partial q}}\frac{1}{\beta\int_Z\left(z-T\left(z\right)\right)\tilde{f}(z)\mathrm{d}z}\int_Z \Theta_{\mathrm{m}}\frac{Q\left(m\right)}{Q^s\left(m\right)}\frac{\partial Q^s\left(m\right)}{\partial q}\mathrm{d}m.\end{aligned} \tag{40}$$

If we take numerate good c as consumption in period one and general good x as consumption in period two in a standard two period model, we could learn from (39) that optimal capital tax is nonzero if misperception q^s and Q^s are both affected by actual price of capital q. Misperception, or more specifically, the influence of interest rate on perceived marginal income tax, is a key assumption which leads to non-zero capital taxation. By contrast, in Saez (2002) and Golosov et al. (2013), the key point becomes heterogeneous tastes on period-2 consumption among consumers.

The sign of linear capital income tax depends on misperception pattern of the whole population. When $\frac{\partial q^s}{\partial q} > 0$, linear capital tax is positive if an increase in capital tax pushes down individuals' perception of marginal income tax rate at most income levels. Otherwise, if an increase in capital tax pushes upward individuals' perception of marginal income tax rate at most income levels, it is optimal to adopt capital subsidy.

VII. Conclusion

This paper explores optimal linear commodity tax mixed with non-linear labor income tax formulas when inattentive agents misperceive commodity prices and marginal income tax rates. To build up the relationships between behavioral agents and optimal tax theory with fully rational individuals, we find as-if rational consumers to characterize inattentive consumers' behavior and give the expressions of modified Roy's identity and Slutsky equation. Since labor supply would be impacted by misperception, we carefully define labor income elasticities by applying an income tax reform function in general form. Based on these efforts, we express optimal tax formulas in measurable sufficient elasticities and misperception wedges using both mechanism design and tax perturbation method. The form of our optimal income tax formula indicates a combination of results in Jacobs and Boadway (2014) and Farhi and Gabaix (2020). The expressions of optimal commodity tax formula from two methods are different but could be proved to be equivalent, leading to a connection between Corlett-Hague rule and many person Ramsey rule. Overall, optimal mixed tax schedule should coordinate the following roles of tax tools: (1) Directly improve redistribution. (2) Directly promote efficiency, including correction for misperception wedges. (3) Cross-effect on the roles of another tax tool.

The implications behind optimal tax formulas are carefully explored. We find that optimal marginal income tax rates at endpoints of the skill distribution are not necessarily zero due to misperception. Our analyze could be integrated into discussion on indexing tax to inflation rate because welfare effects of nominal illusion worth consideration when government wants to use tax indexation to improve the effectiveness of fiscal policies. In addition, we underscore the role of indirect tax under typical preference structure for Atkinson and Stiglitz theorem. Uniform tax rule is not supported when consumers misperceive commodity prices. This finding corresponds to the idea in Boccanfuso and Ferey (2019) that inattention creates incentives for government to use discretionary policies.

Our work focus on price misperception as a special kind of inattention, which could be parallel to the topic in Allcott, Lockwood and Taubinsky (2019) as they explore optimal mixed taxation with wedge between "experienced" and "decision" utility. But we emphasis cross-effects of misperception, which cannot be captured by a single wedge between utilities. The effect of commodity prices on perception of marginal income tax rate is crucial to the role of commodity tax. Still quantitative analysis should be used to bridge theory and practice. In future work, we will focus on specific kinds of goods and types of misperception, and use calibration and simulation to make quantitative analysis.

Appendix

Appendix A gives more details on individual's behavior under misperceived prices with the help of decision pattern of as-if rational consumers. Appendix B provides derivation procedure of elasticities expressed with derivatives of indirect utility function using shift function method. Elasticities in Saez's form are also defined, and connections between the two class of elasticities are built. Appendix C provides tax perturbation method to get optimal mixed tax formula. Appendix D contains proofs not included in the main paper.

Appendix A Complements on individual's behavior under misperceived prices

A.1 Properties of individual's behavior in stage 2

Properties of conditional demand function. Take partial derivatives of bounded rational agent's budget constraint $c_n + \sum_i q_i x_{in} = y_n$ on y_n z_n and q_j separately to get

$$\frac{\partial c_n^s}{\partial z_n} = -\sum_i q_i \frac{\partial x_{in}^s}{\partial z_n}; \frac{\partial c_n^s}{\partial y_n} + \sum_i q_i \frac{\partial x_{in}^s}{\partial y_n} = 1; \frac{\partial c_n^s}{\partial q_j} + \sum_i q_i \frac{\partial x_{in}^s}{\partial q_j} + x_{jn}^s = 0. \tag{A.1}$$

Take derivatives of as-if rational agent's budget constraint $c_n^r + \sum_i q_i^s x_{in}^r = \bar{y}_n (q, q^s, y_n, z_n/n)$ on $\bar{y}_n$, z_n and q_j separately to get

$$\frac{\partial c_n^r}{\partial z_n} + \sum_i q_i^s \frac{\partial x_{in}^r}{\partial z_n} = 0; \frac{\partial c_n^r}{\partial \bar{y}_n} + \sum_i q_i^s \frac{\partial x_{in}^r}{\partial \bar{y}_n} = 1; \frac{\mathrm{d} c_n^r}{\mathrm{d} q_j} + \sum_i q_i^s \frac{\mathrm{d} x_{in}^r}{\mathrm{d} q_j} + \sum_i x_{in}^r \frac{\mathrm{d} q_i^s}{\mathrm{d} q_j} = \frac{\mathrm{d} \bar{y}_n}{\mathrm{d} q_j}. \tag{A.2}$$

Since $c_n^s = c_n^r, x_n^s = x_n^r$, we have

$$\frac{\partial x_{in}^s}{\partial y_n} = \frac{\partial x_{in}^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial y_n}; \frac{\partial x_{in}^s}{\partial z_n} = \frac{\partial x_{in}^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial z_n} + \frac{\partial x_{in}^r}{\partial z_n}; \frac{\partial x_{in}^s}{\partial q_j} = \frac{\partial x_{in}^r}{\partial \bar{y}_n} \frac{\mathrm{d} \bar{y}_n}{\mathrm{d} q_j} + \frac{\mathrm{d} x_{in}^r}{\mathrm{d} q_j}. \tag{A.3}$$

Equations in (A.3) build up relationships between (A.1) and (A.2).

Properties of conditional indirect utility function. For as-if rational consumers, we feel free to use envelop theorem in his utility-maximization problem described in (2) to get

$$\frac{\partial v_n^r}{\partial \bar{y}_n} = u_c; \frac{\partial v_n^r}{\partial z_n} = \frac{1}{n} u_l; \frac{\partial v_n^r}{\partial q_k^s} = -u_c x_{kn}^r, \forall k \in \{1, \ldots, I\}. \tag{A.4}$$

Since $v_n^s = v_n^r$, we have the following links between derivatives of indirect utilities of behavioral and as-if rational consumers:

$$\frac{\partial v_n^s}{\partial y_n} = \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial y_n}; \frac{\partial v_n^s}{\partial z_n} = \frac{\partial v_n^r}{\partial z_n} + \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial z_n}; \frac{\partial v_n^s}{\partial q_i} = \frac{\partial v_n^r}{\partial q_i} + \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\mathrm{d} \bar{y}_n}{\mathrm{d} q_i}. \tag{A.5}$$

Properties of conditional expenditure function. For as-if rational consumers, we feel free to use envelop theorem in his expenditure-minimizing problem described in (5) to get

$$\frac{\partial c_n^{r*}}{\partial z_n} + \sum_i q_i^s \frac{\partial x_{in}^{r*}}{\partial z_n} = \frac{\partial e_n^r}{\partial z_n} = -\frac{u_l/n}{u_c}; \tag{A.6}$$

$$\frac{\partial c_n^{r*}}{\partial v_n^r} + \sum_i q_i^s \frac{\partial x_{in}^{r*}}{\partial v_n^r} = \frac{\partial e_n^r}{\partial v_n^r} = \frac{1}{u_c}; \tag{A.7}$$

$$x_{jn}^r + \sum_i q_i^s \frac{\partial x_{in}^{r*}}{\partial q_j^s} + \frac{\partial c_n^{r*}}{\partial q_j^s} = \frac{\partial e_n^r}{\partial q_j^s} = x_{jn}^r. \tag{A.8}$$

Then we could use (A.12) to characterize properties of e_n^s. The definition of e_n^s is given in (6). Take partial derivatives of e_n^s on z_n, v_n and q_j separately, we have

$$\frac{\partial e_n^s}{\partial z_n} = \frac{\partial c_n^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial z_n} + \sum_i q_i^s \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial z_n} + \sum_i \left(q_i - q_i^s\right) \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial z_n}; \tag{A.9}$$

$$\frac{\partial e_n^s}{\partial z_n} = \frac{\partial c_n^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial z_n} + \sum_i q_i^s \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial z_n} + \sum_i \left(q_i - q_i^s\right) \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial z_n}; \tag{A.10}$$

$$\begin{aligned}
\frac{\partial e_n^s}{\partial q_j} &= \sum_k \frac{\partial c_n^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + \sum_i q_i \sum_k \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + x_{jn}^{r*} \\
&= \sum_k \left(\frac{\partial c_n^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + \sum_i q_i^s \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} \right) \\
&\quad + \sum_i \left(q_i - q_i^s\right) \sum_k \frac{\partial x_{in}^{r*}\left(q^s, v_n^s, z_n/n\right)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + x_{jn}^{r*}.
\end{aligned} \tag{A.11}$$

Links between derivatives of compensated and uncompensated conditional demand. Combine (A.3) with (A.6) to (A.8) to get links between derivatives of compensated and uncompensated conditional demand.

$$\begin{aligned}\frac{\partial x_n^{r*}}{\partial v_n^r} &= \frac{1}{u_c}\frac{\partial x_{in}^r}{\partial \bar{y}_n};\\ \frac{\partial x_{in}^{r*}}{\partial z_n} &= -\frac{u_l/n}{u_c}\frac{\partial x_n^r}{\partial \bar{y}_n} + \frac{\partial x_n^r}{\partial z_n};\\ \frac{\partial x_{in}^{r*}}{\partial q_k^s} &= \frac{\partial x_{in}^r}{\partial q_k^s} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} x_{kn}^r.\end{aligned} \tag{A.12}$$

A.2 Properties of individual's behavior in stage 1

Properties of conditional indirect utility function. Use (4), we could transform (9) into

$$\left(\frac{\partial v_n^r}{\partial \bar{y}_n}\frac{\partial \bar{y}_n}{\partial z_n} + \frac{\partial v_n^r}{\partial z_n}\right) / \left(\frac{\partial v_n^r}{\partial \bar{y}_n}\frac{\partial \bar{y}_n}{\partial y_n}\right) = -Q_n^s. \tag{A.13}$$

Combine it with (A.4) to get

$$\frac{u_l/n}{u_c}\left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1} + \left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}\frac{\partial \bar{y}_n}{\partial z_n} = -Q_n^s. \tag{A.14}$$

Properties of unconditional indirect utility function. We first apply envelop theorem to as-if rational agent's first-stage maximization problem to get

$$\frac{\partial V_n^r}{\partial q_j} = \frac{\partial v_n^s}{\partial q_j}; \frac{\partial V_n^r}{\partial \bar{R}_n} = \frac{\partial v_n^s}{\partial y}; \frac{\partial V_n^r}{\partial Q_n^s} = \frac{\partial v_n^s}{\partial y} z_n. \tag{A.15}$$

Since $z_n(q, Q_n, \mathbf{Q}, R_n) = z_n^r(q, Q_n^s(q, Q_n, \mathbf{Q}), \bar{R}_n(q, Q_n, Q_n^s, R_n))$, we get relationships between derivatives of z_n and z_n^r as:

$$\begin{aligned}\frac{\partial z_n}{\partial R_n} &= \frac{\partial z_n^r}{\partial \bar{R}_n}\frac{\partial \bar{R}_n}{\partial R_n};\\ \frac{\mathrm{d}z_n}{\mathrm{d}Q_n} &= \frac{\partial z_n^r}{\partial Q_n^s}\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} + \frac{\partial z_n^r}{\partial \bar{R}_n}\frac{\mathrm{d}\bar{R}_n}{\mathrm{d}Q_n};\\ \frac{\mathrm{d}z_n}{\mathrm{d}q_j} &= \frac{\mathrm{d}z_n^r}{\mathrm{d}q_j} = \frac{\partial z_n^r}{\partial q_n} + \frac{\partial z_n^r}{\partial Q_n^s}\frac{\partial Q_n^s}{\partial q_j} + \frac{\partial z_n^r}{\partial \bar{R}_n}\frac{\mathrm{d}\bar{R}_n}{\mathrm{d}q_j}.\end{aligned} \tag{A.16}$$

Since $\bar{R}_n(q, Q_n, Q_n^s, R_n) = R_n - (Q_n^s - Q_n)\, z_n^r(q, Q_n^s, \bar{R}_n(q, Q_n, Q_n^s, R_n))$, we get derivatives of $\bar{R}_n$

as

$$\begin{aligned}
\frac{\partial \bar{R}_n}{\partial R_n} =& 1-\frac{Q_n^s-Q_n}{Q_n}\frac{\partial z_n}{\partial R_n}Q_n=\left(1+\left(Q_n^s-Q_n\right)\frac{\partial z_n^r}{\partial \bar{R}_n}\right)^{-1};\\
\frac{\mathrm{d}\bar{R}_n}{\mathrm{d}q_j} =& -\left[\left(Q_n^s-Q_n\right)\frac{\partial z_n}{\partial q_j}+z_n^r\frac{\partial Q_n^s}{\partial q_j}\right];\\
\frac{\mathrm{d}\bar{R}_n}{\mathrm{d}Q_n} =& -\left[\left(Q_n^s-Q_n\right)\frac{\partial z_n^r}{\partial Q_n^s}\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}+z_n\left(\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}-1\right)\right]\frac{\partial \bar{R}_n}{\partial R_n}.
\end{aligned} \tag{A.17}$$

Since $V_n^s(q, Q_n, \mathbf{Q}, R_n) = V_n^r(q, Q_n^s, \bar{R}_n(q, Q_n, Q_n^s, R_n))$, with the derivatives defined above, we have

$$\begin{aligned}
\frac{\partial V_n^s}{\partial R_n} =& \frac{\partial v_n^s}{\partial y}\frac{\partial \bar{R}_n}{\partial R_n};\\
\frac{\partial V_n^s}{\partial q_j} =& \frac{\partial v_n^s}{\partial q_j}-\left(Q_n^s-Q_n\right)\frac{\partial z_n}{\partial q_j}\frac{\partial v_n^s}{\partial y};\\
\frac{\partial V_n^s}{\partial Q_n} =& \left[z_n\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}-z_n\left(\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}-1\right)\frac{\partial \bar{R}_n}{\partial R_n}-\left(Q_n^s-Q_n\right)\frac{\partial z_n^r}{\partial Q_n^s}\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}\frac{\partial \bar{R}_n}{\partial R_n}\right]\frac{\partial v_n^s}{\partial y};\\
\frac{\partial V_n^s}{\partial Q_m} =& \left(Q_n^s-Q_n\right)\left(z_n\frac{\partial z_n^r}{\partial \bar{R}_n}-\frac{\partial z_n^r}{\partial Q_n^s}\right)\frac{\partial \bar{R}_n}{\partial R_n}\frac{\partial v_n^s}{\partial y}\frac{\partial Q_n^s}{\partial Q_m}.
\end{aligned} \tag{A.18}$$

Appendix B Derivation of elasticities

B.1 Elasticities using shift function method

Since the shift function of a labor tax reform is defined as

$$L(z_n, z, n, \phi, q) \equiv n\tilde{Q}_n^s(q, \tilde{Q}_n, \tilde{\mathbf{Q}}, R_0)v_y^s\left(q, \tilde{y}_n, z_n/n\right)+v_l^s\left(q, \tilde{y}_n, z_n/n\right).$$

Then the foc at $\psi = 0$ is

$$\begin{aligned}
\frac{\mathrm{d}L(z_n, z, n, 0, q)}{\mathrm{d}z_n} =& nv_y^s\left(\frac{\partial Q_n^s}{\partial Q_n}+\frac{\partial Q_n^s}{\partial \mathbf{Q}}\cdot\delta_n\right)Q'\left(z_n\right)+nv_{yy}^sQ_n^sQ_n+nQ_n^sv_{yz}+nQ_nv_{yz}+nv_{zz};\\
\frac{\partial L(z_n, z, n, 0, q)}{\partial n} =& Q_n^sv_y^s+nQ_n^s\left(-\frac{z}{n^2}\right)v_{yl}^s+\left(-\frac{z}{n^2}\right)v_{ll}^s=-\left[\left(v_{zz}^s+v_{zy}^sQ_n^s\right)z_n+v_z^s\right];\\
\frac{\partial L(z_n, z, n, 0, q)}{\partial \phi} =& -nv_y^s\left(\frac{\partial Q_n^s}{\partial Q_n}+\frac{\partial Q_n^s}{\partial \mathbf{Q}}\cdot\delta_n\right)\tau'\left(z_n\right)-\tau\left(z\right)nQ_n^sv_{yy}^s-\tau\left(z\right)nv_{zy}^s\\
&-nv_y^s\int_N\tau'(z_m)\frac{\partial Q_n^s}{\partial \mathbf{Q}}\left(1-\delta_n\right)\mathrm{d}m+nv_y^s\int_N Q'(z_m)\frac{\partial Q_n^s}{\partial \mathbf{Q}}\frac{\mathrm{d}z_m}{\mathrm{d}\phi}\left(1-\delta_n\right)\mathrm{d}m;\\
\frac{\partial L(z_n, z, n, 0, q)}{\partial q_j} =& nQ_n^s(q, Q_n, \mathbf{Q}, R_0)v_{yq_j}^s+nv_{zq_j}^s\left(q, y_n, z_n/n\right)+nv_y^s\frac{\partial Q_n^s}{\partial q_j}.
\end{aligned}$$

Then we could use implicit function theorem to express elasticities of labor income in derivatives of indirect utility function, which is useful in deriving optimal tax formula under mechanism design. For example, we have $\varepsilon_{zn} = \frac{n}{z}\frac{\partial z}{\partial n} = -\frac{n}{z}\frac{\mathrm{d}L(z_n,z,n,0)/\mathrm{d}z_n}{\partial L(z_n,z,n,0)/\partial n}$. These elasticities are different from Saez (2001) since they account for nonlinearity of income tax schedule. In the next part, we return to Seaz's form taken into consideration individual's misperception.

B.2 Elasticities in Saez's form

Following Saez (2001), define income elasticity of z_n as

$$\eta = Q_n \frac{\partial z_n}{\partial R_n}. \tag{B.1}$$

Note that η is different from ϵ_z^I defined in the main text because the later accounts for the "circular process" caused by nonlinearity of tax schedule (Jacquet et al., 2013). To define tax elasticities of z_n, observe that

$$\begin{aligned}
\frac{\mathrm{d}z_n}{\mathrm{d}Q_n} &= \frac{\partial z_n^r}{\partial Q_n^s}\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} - \frac{\partial z_n^r}{\partial \bar{R}_n}\left[(Q_n^s - Q_n)\frac{\partial z_n^r}{\partial Q_n^s}\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} + z_n\left(\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} - 1\right)\right]\frac{\partial \bar{R}_n}{\partial R_n} \\
&= \frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}\left(\frac{\partial z_n^r}{\partial Q_n^s}\frac{\partial \bar{R}_n}{\partial R_n} - \frac{z_n}{Q_n}\eta\right) + \frac{z_n}{Q_n}\eta.
\end{aligned}$$

Then we define uncompensated tax elasticity of labor income ξ_{zQ}^s and compensated tax elasticity of labor income ξ_{zQ}^{cs} as

$$\begin{aligned}
\xi_{zQ}^s &\equiv \frac{\mathrm{d}z_n}{\mathrm{d}Q_n}\frac{Q_n}{z_n} = \frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}\left(\frac{\partial z_n^r}{\partial Q_n^s}\frac{Q_n}{z_n}\frac{\partial \bar{R}_n}{\partial R_n} - \eta\right) + \eta; \\
\xi_{zQ}^{cs} &\equiv \xi_{zQ}^s - \eta = \frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n}\left(\frac{\partial z_n^r}{\partial Q_n^s}\frac{Q_n}{z_n}\frac{\partial \bar{R}_n}{\partial R_n} - \eta\right).
\end{aligned} \tag{B.2}$$

Again, ξ_{zQ}^{cs} is different from ε_{zQ}^* for not accounting for "circular process".

Similarly, observe that

$$\begin{aligned}
\frac{\mathrm{d}z_n}{\mathrm{d}Q_m} &= \frac{\partial z_n^r}{\partial Q_n^s}\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_m} - \frac{\partial z_n^r}{\partial \bar{R}_n}\left[(Q_n^s - Q_n)\frac{\partial z_n^r}{\partial Q_n^s}\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_m} + z_n\frac{\partial Q_n^s}{\partial Q_m}\right]\frac{\partial \bar{R}_n}{\partial R_n} \\
&= \frac{\partial z_n^r}{\partial Q_n^s}\frac{\partial \bar{R}_n}{\partial R_n}\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_m} - z_n\frac{\partial Q_n^s}{\partial Q_m}\frac{\eta}{Q_n}.
\end{aligned}$$

We could define compensated tax elasticity when marginal tax rate at z_m changes as

$$\xi^s_{Qm} \equiv \frac{\mathrm{d}z_n}{\mathrm{d}Q_m}\frac{Q_n}{z_n} = \frac{\partial Q^s_n}{\partial Q_m}\left(\frac{\partial z^r_n}{\partial Q^s_n}\frac{Q_n}{z_n}\frac{\partial \bar{R}_n}{\partial R_n} - \eta\right). \tag{B.3}$$

B.3 Connection between two sets of income elasticities

We connect previous two sets of income elasticities by expressing impact of an income tax reform on z_n with income elasticities of Saez's form. Define the following income tax reform as

$$\tilde{T}(z,\phi) = T(z) + \phi\tau(z), \tag{B.4}$$

in which ϕ is policy change parameter and $\tau(z)$ is a random policy change function. Individual's labor income and indirect utility after the reform is

$$z_n(q, \tilde{Q}_n, \tilde{\mathbf{Q}}, \tilde{R}_n); V^s_n(q, \tilde{Q}_n, \tilde{\mathbf{Q}}, \tilde{R}_n),$$

in which marginal retention rate after the tax reform is

$$\tilde{Q}(z_n) = Q(z) - \phi\tau'(z),$$

and generalized revenue after the tax reform is

$$\tilde{R}_n = z - \tilde{T}(z,\phi) - \tilde{Q}_n z = zT'(z) - T(z) - \phi\tau(z) + z\phi\tau'(z).$$

Therefore, impact of the tax reform on z_n could be expressed as

$$\begin{aligned}\frac{\mathrm{d}z_n(q, \tilde{Q}_n, \tilde{\mathbf{Q}}, \tilde{R}_n)}{\mathrm{d}\phi} =& \frac{\mathrm{d}z_n}{\mathrm{d}Q_n}\frac{\mathrm{d}\tilde{Q}_n}{\mathrm{d}\phi} + \frac{\partial z_n}{\partial R_n}\frac{\mathrm{d}\tilde{R}_n}{\mathrm{d}\phi} + \int_N \frac{\partial z_n}{\partial Q_m}\frac{\mathrm{d}\tilde{Q}_m}{\mathrm{d}\phi}(1-\delta_n)\,dm \\ =& \left(-\frac{dz_n}{dQ_n} + z_n\frac{\partial z_n}{\partial R_n}\right)\left(T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}\phi} + \tau'(z_n)\right) - \tau(z_n)\frac{\partial z_n}{\partial R_n} \\ & - \int_N \frac{\partial z_n}{\partial Q_m}\left(T''(z_m)\frac{\mathrm{d}z_m}{\mathrm{d}\phi} + \tau'(z_m)\right)(1-\delta_n)\,\mathrm{d}m\end{aligned} \tag{B.5}$$

Use elasticities defined in section B, we could transform the equation above into

$$\frac{\mathrm{d}z_n}{\mathrm{d}\phi}\left(1+T''(z_n)\frac{z_n}{Q_n}\xi_{zQ}^{cs}\right) = -\frac{z_n}{Q_n}\left[\tau'(z_n)\xi_{zQ}^{cs} + \frac{\tau(z_n)}{z_n}\eta + \int_N \xi_{Qm}^{s}\left(T''\left(z_m\right)\frac{\mathrm{d}z_m}{\mathrm{d}\phi} + \tau'(z_m)\right)\left(1-\delta_n\right)\mathrm{d}m\right].$$

By defining

$$\tilde{\xi}_{zQ}^{cs} \equiv \frac{\xi_{zQ}^{cs}}{1+T''\left(z_n\right)\frac{z_n}{Q_n}\xi_{zQ}^{cs}}; \tilde{\eta} \equiv \frac{\eta}{1+T''\left(z_n\right)\frac{z_n}{Q_n}\xi_{zQ}^{cs}}; \tilde{\xi}_{Qm}^{s} \equiv \frac{\xi_{Qm}^{s}}{1+T''\left(z_n\right)\frac{z_n}{Q_n}\xi_{zQ}^{cs}},$$

we finally get

$$\frac{\mathrm{d}z_n}{\mathrm{d}\phi} = -\frac{z_n}{Q_n}\left[\tau'(z_n)\tilde{\xi}_{zQ}^{cs} + \frac{\tau(z_n)}{z_n}\tilde{\eta} + \int_N \tilde{\xi}_{Qm}^{s}\left(\tau'(z_m) - Q'\left(z_m\right)\frac{\mathrm{d}z_m}{\mathrm{d}\phi}\right)\left(1-\delta_n\right)dm\right].$$

Compared with (15), we have the following relationships between elasticities:

$$\tilde{\xi}_{zQ}^{cs} = -\varepsilon_{zQ}^{*}; \tilde{\eta} = -\varepsilon_{z}^{I}; \tilde{\xi}_{Qm}^{s} = -\varepsilon_{zQm}^{*}. \tag{B.6}$$

The signs of two kinds of elasticities are opposite because we define tax elasticities of labor income through changes in ϕ in shift function method, while elasticities in Saez's form are defined through changes in marginal retention rate. The directions of impacts of ϕ and Q_n are opposite.

Appendix C Derive optimal taxation by tax perturbation method

In this section, we use tax perturbation method to derive optimal tax formulas. The first part is also the proof of proposition 3.

C.1 Optimal commodity tax

Changes in commodity tax influences both the social welfare and government's tax revenue by changing individual's behavior. The government's problem has been described in the main text, which requires that $\frac{\mathrm{d}W}{\mathrm{d}q_j}/\mu + \frac{\mathrm{d}B}{\mathrm{d}q_j} = 0$ for $\forall j \in \{1:I\}$. The expression of $\frac{\mathrm{d}W}{\mathrm{d}q_j}/\mu + \frac{\mathrm{d}B}{\mathrm{d}q_j}$ is given in (27). We make the following transformations in order to express $\frac{\mathrm{d}W}{\mathrm{d}q_j}/\mu + \frac{\mathrm{d}B}{\mathrm{d}q_j}$ with behavioral elasticities.

According to components of (27), we firstly rewrite $\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n}$ as

$$\begin{aligned}\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n} =& \frac{\partial V_n^s/\partial q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n} + \frac{\mathrm{d}R_n}{\mathrm{d}q_j} + \frac{\partial V_n^s/\partial Q_n}{\mathrm{d}V_n^s/\mathrm{d}R_n}\frac{\mathrm{d}Q_n}{\mathrm{d}q_j} + \int_N \frac{\partial V_n^s/\partial Q_m}{\mathrm{d}V_n^s/\mathrm{d}R_n}\frac{\mathrm{d}Q_m}{\mathrm{d}q_j}\mathrm{d}m \\ =& \frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} - (Q_n^s - Q_n)\frac{\partial z_n}{\partial q_j}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} \\ &+ z_n T''(z_n)\xi_Q^{cs}\frac{Q_n^s - Q_n}{Q_n}\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} \\ &- z_n\frac{(Q_n^s - Q_n)}{Q_n}\int_N \xi_{Qm}^s\frac{\mathrm{d}Q_m}{\mathrm{d}q_j}\mathrm{d}m\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1}.\end{aligned}$$

Since

$$\begin{aligned}\frac{\mathrm{d}z_n}{\mathrm{d}q_j} - \int_N \frac{\partial z_n}{\partial Q_m}\frac{\mathrm{d}Q_m}{\mathrm{d}q_j}\mathrm{d}m - \frac{\partial z_n}{\partial q_j} =& \frac{\partial z_n}{\partial Q_n}\frac{\mathrm{d}Q_n}{\mathrm{d}q_j} + \frac{\partial z_n}{\partial R_n}\frac{\mathrm{d}R_n}{\mathrm{d}q_j} \\ =& -\xi_Q^s\frac{z_n}{Q_n}T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j} + \eta\frac{z_n}{Q_n}T''(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j} \\ =& -T''(z_n)\xi_Q^{cs}\frac{z_n}{Q_n}\frac{\mathrm{d}z_n}{\mathrm{d}q_j},\end{aligned} \tag{C.1}$$

we therefore simplify $\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n}$ into

$$\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n} = \frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} - (Q_n^s - Q_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1}. \tag{C.2}$$

Secondly, we need to use elasticities to express the impact of q_i on commodity demand. $\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j}$ could be decomposed into

$$\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j} = \frac{\partial x_{in}^s}{\partial q_j} + \frac{\partial x_{in}^s}{\partial y_n}\frac{\mathrm{d}y_n}{\mathrm{d}q_j} + \frac{\partial x_{in}^s}{\partial z_n}\frac{\mathrm{d}z_n}{\mathrm{d}q_j}.$$

Since $\frac{\mathrm{d}y_n}{\mathrm{d}q_j} = Q_n\frac{\mathrm{d}z_n}{\mathrm{d}q_j}$, we use modified Slutsky equation (8) as well as Roy's identity (7) to get

$$\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j} = \sum_k \frac{\partial x_{in}^{r*}}{\partial q_i^s}\frac{\partial q_k^s}{\partial q_j} - \frac{\partial x_{in}^s}{\partial y_n}\left(w_{jn}^q + x_{jn}^{r*}\right) + \left[(Q_n - Q_n^s)\frac{\partial x_{in}^s}{\partial y_n} + \frac{\partial x_{in}^{r*}}{\partial z_n}\right]\frac{\mathrm{d}z_n}{\mathrm{d}q_j}. \tag{C.3}$$

Then we could transform (27) by substituting $\frac{\mathrm{d}V_n^s/\mathrm{d}q_j}{\mathrm{d}V_n^s/\mathrm{d}R_n}$ and $\frac{\mathrm{d}x_{in}^s}{\mathrm{d}q_j}$ with expression in (C.2) and (C.3). Using the definition that $g_n \equiv \frac{\Psi' v_y^s}{\mu}$ and $\xi_{zq_j} \equiv \frac{q_j}{z_n}\frac{\mathrm{d}z_n}{\mathrm{d}q_j}$ and the relationship $\int_n^{\bar{n}} af(n)\mathrm{d}n = \int_z^{\bar{z}} a\tilde{f}(z)\mathrm{d}z$

implied by $F(n) \equiv \tilde{F}(z_n)$, we could express the impact of commodity tax on net social welfare as

$$\begin{aligned}\frac{\mathrm{d}W}{\mathrm{d}q_j}/\mu+\frac{\mathrm{d}B}{\mathrm{d}q_j}=&\int_Z\sum_i t_i\left(\sum_k\frac{\partial x_{in}^{r*}}{\partial q_i^s}\frac{\partial q_k^s}{\partial q_j}\right)\tilde{f}(z)\mathrm{d}z-\int_Z w_{jn}^q\tilde{f}(z)\mathrm{d}z\\&+\int_Z\left(1-g_n-\sum_i t_i\frac{\partial x_{in}^s}{\partial y_n}\right)\left(w_{jn}^q+x_{jn}^{r*}\right)\tilde{f}(z)\mathrm{d}z\\&-\int_Z\left(Q_n^s-Q_n\right)\left(g_n+\sum_i t_i\frac{\partial x_{in}^s}{\partial y_n}\right)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\tilde{f}(z)\mathrm{d}z\\&+\int_Z\left[\sum_i t_i\frac{\partial x_{in}^{r*}}{\partial z_n}+T'(z_n)\right]\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\tilde{f}(z)\mathrm{d}z.\end{aligned}$$

Applying $\frac{\mathrm{d}W}{\mathrm{d}q_j}/\mu+\frac{\mathrm{d}B}{\mathrm{d}q_j}=0$ gives optimal linear commodity tax formula (28).

C.2 Optimal income tax

Due to the nonlinearity of income taxation, we could not directly perturb $Q(z_n)$ to derive optimal income tax formula. So we assume an income tax reform as in (B.4) and adopt the idea of calculus of variations to get optimal income tax function. The tax reform leads to the following changes in marginal retention rate, generalized revenue and disposable income:

$$\tilde{Q}_n=1-T'(z_n)-\phi\tau'(z_n);\tilde{R}_n=z_n\left(T'(z_n)+\phi\tau'(z_n)\right)-T(z_n)-\phi\tau(z_n);\tilde{y}_n=z_n-T(z_n)-\phi\tau(z_n).$$

The tilde over a variable indicates that it is realized after the tax reform. At $\phi=0$, the marginal impacts on income tax, marginal retention rate and generalized revenue are

$$\frac{\mathrm{d}\tilde{Q}_n}{\mathrm{d}\phi}=-T''\left(z_n\right)\frac{\mathrm{d}z_n}{\mathrm{d}\phi}-\tau'(z_n); \tag{C.4}$$

$$\frac{\mathrm{d}\tilde{R}_n}{\mathrm{d}\phi}=\left(T''\left(z_n\right)\frac{\mathrm{d}z_n}{\mathrm{d}\phi}+\tau'(z_n)\right)z_n-\tau(z_n); \tag{C.5}$$

$$\frac{\mathrm{d}\tilde{T}(z_n)}{\mathrm{d}\phi}=\tau(z_n)+T'\left(z_n\right)\frac{\mathrm{d}z_n}{\mathrm{d}\phi}. \tag{C.6}$$

The reform also has the following marginal influence on individual's commodity demand:

$$\frac{\mathrm{d}x_{in}^s\left(q,\tilde{y},z_n/n\right)}{\mathrm{d}\phi}=\frac{\partial x_{in}^s}{\partial y_n}\frac{\mathrm{d}y_n}{\mathrm{d}\phi}+\frac{\partial x_{in}^s}{\partial z_n}\frac{\mathrm{d}z_n}{\mathrm{d}\phi}-\frac{\partial x_{in}^s}{\partial y_n}\tau(z)=\frac{x_{in}^s}{z_n}\varepsilon_{x_iz}\frac{\mathrm{d}z_n}{\mathrm{d}\phi}-\frac{\partial x_{in}^s}{\partial y_n}\tau(z), \tag{C.7}$$

and on indirect utility function:

$$\begin{aligned}\frac{\mathrm{d}\tilde{V}_n^s}{\mathrm{d}\phi} =&\frac{\partial V_n^s}{\partial \tilde{Q}_n}\frac{\mathrm{d}\tilde{Q}_n}{\mathrm{d}\phi}+\frac{\partial V_n^s}{\partial \tilde{R}_n}\frac{\mathrm{d}\tilde{R}_n}{\mathrm{d}\phi}+\int_N \frac{\partial V_n^s}{\partial Q_m}\frac{\mathrm{d}Q_m}{\mathrm{d}\phi}\left(1-\delta_n\right)dm \\ =&-\tau(z_n)\frac{\partial V_n^s}{\partial R_n}+\frac{\partial V_n^s}{\partial R_n}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} z_n\tau_n^b\xi_{zQ}^{cs}\left(T''\left(z_n\right)\frac{\mathrm{d}z_n}{\mathrm{d}\phi}+\tau'(z_n)\right) \\ &-\frac{\partial V_n^s}{\partial R_n}\left(\frac{\partial \bar{R}_n}{\partial R_n}\right)^{-1} z_n\tau_n^b\int_N \xi_{Qm}^s\frac{\mathrm{d}Q_m}{\mathrm{d}\phi}\left(1-\delta_n\right)\mathrm{d}m.\end{aligned} \tag{C.8}$$

The impact on labor income z_n has been presented in (B.5).

As for the government's problem, since changes in ϕ influences both the social welfare and government's tax revenue by changing individual's behavior. The social welfare function after the reform is

$$\tilde{W} \equiv \int_N \Psi\left(V_n^s(q,\tilde{Q}_n,\tilde{\mathbf{Q}},\tilde{R}_n)\right) f(n)\mathrm{d}n,$$

and government's revenue after the reform is

$$\tilde{B} = \int_N \tilde{T}(z_n)f\left(n\right)\mathrm{d}n+\int_N \sum_i t_i x_{in}^s\left(q,\tilde{y},z_n/n\right)f\left(n\right)\mathrm{d}n.$$

The government's problem is to choose optimal ϕ to maximize net social welfare function $\tilde{W}/\mu-\tilde{B}$. The marginal influence of ϕ on net social welfare is

$$\begin{aligned}\frac{\mathrm{d}\tilde{W}}{\mathrm{d}\phi}/\mu+\frac{\mathrm{d}\tilde{B}}{\mathrm{d}\phi} =&\int \frac{\Psi'(\tilde{V}_n^s)}{\mu}\frac{\partial \tilde{V}_n^s}{\partial \tilde{R}_n}\frac{\mathrm{d}\tilde{V}_n^s/\mathrm{d}\phi}{\partial \tilde{V}_n^s/\partial \tilde{R}_n}f\left(n\right)dn \\ &+\int \frac{\mathrm{d}\tilde{T}(z_n)}{\mathrm{d}\phi}f\left(n\right)\mathrm{d}n+\int \sum_i t_i\frac{\mathrm{d}x_{in}^s\left(q,\tilde{y},z_n/n\right)}{\mathrm{d}\phi}f\left(n\right)\mathrm{d}n.\end{aligned}$$

Optimal income tax requires that marginal influence of ϕ at $\phi=0$ be zero. Substituting (C.4) to (C.8) into $\frac{\mathrm{d}\tilde{W}}{\mathrm{d}\phi}/\mu+\frac{\mathrm{d}\tilde{B}}{\mathrm{d}\phi}=0$ yields

$$\begin{aligned}&\int\left(1-Q_n+\sum_i t_i\frac{x_{in}^s}{z_n}\varepsilon_{x_iz}\right)\frac{\mathrm{d}z_n}{\mathrm{d}\phi}f\left(n\right)\mathrm{d}n \\ =&-\int \tau(z_n)\left(1-g_n-\sum_i t_i\frac{\partial x_{in}^s}{\partial y_n}\right)f\left(n\right)\mathrm{d}n-\int g_n\tau_n^b\eta\tau(z_n)f\left(n\right)\mathrm{d}n \\ &-\int g_nz_n\tau_n^b\left[\xi_{zQ}^{cs}\left(T''\frac{\mathrm{d}z_n}{\mathrm{d}\phi}+\tau'(z_n)\right)+\int \xi_{Qm}^s\left(T''\left(z_m\right)\frac{\mathrm{d}z_m}{\mathrm{d}\phi}+\tau'(z_m)\right)\left(1-\delta_n\right)dm\right]f\left(n\right)\mathrm{d}n.\end{aligned} \tag{C.9}$$

Since (B.5) could be transformed into

$$\frac{z_n}{Q_n}\left[\xi^{cs}_{zQ}\left(T''\left(z_n\right)\frac{\mathrm{d}z_m}{\mathrm{d}\phi}+\tau'(z_n)\right)+\int\xi^{s}_{Qm}\left(T''\left(z_m\right)\frac{\mathrm{d}z_m}{\mathrm{d}\phi}+\tau'(z_m)\right)(1-\delta_n)\,\mathrm{d}m\right]=-\frac{\tau(z_n)}{Q_n}\eta_n-\frac{\mathrm{d}z_n}{\mathrm{d}\phi},$$

we could simplify (C.9) into

$$\begin{aligned}
&\int_N J_n z_n \varepsilon^*_{zQ}\tau'(z_n)f\left(n\right)\mathrm{d}n \\
=&-\int_N\left[1-g_n-\sum_i t_i\frac{\partial x^s_{in}}{\partial y_n}-J_n\varepsilon^I_z\right]\tau(z_n)f\left(n\right)\mathrm{d}n \\
&-\int_N J_n\left[z_n\int\varepsilon^*_{zQm}\left(T''\left(z_m\right)\frac{dz_m}{d\kappa}+\tau'(z_m)\right)(1-\delta_n)\,dm\right]f\left(n\right)\mathrm{d}n,
\end{aligned}\tag{C.10}$$

in which J_n is defined as

$$J_n\equiv\frac{1-Q_n}{Q_n}+\sum_i\frac{t_i}{Q_n}\frac{x^s_{in}}{z_n}\varepsilon_{x_iz}-g_n\tau^b_n.$$

To solve J_n, we apply a specific tax reform as $\tau(z_n)=1$ if $z_n\geq z^*$, $\tau(z_n)=0$ if $z_n<z^*$, leading to $\tau'(z^*)\mathrm{d}\phi=\mathrm{d}\phi/\mathrm{d}z$ at $z=z^*$. Together with (B.5) and (C.10), the specific tax reform yields solution for J_n at $z_n=z^*$, which is equivalent in optimal nonlinear income tax formula in (35) derived using mechanism design approach.

Appendix D Proofs

D.1 Proof of lemma 1

Proof. Equations in (A.3) build up relationships between (A.1) and (A.2). Therefore, we have

$$\sum_i\left(q^s_i-q_i\right)\frac{\partial x^r_{in}}{\partial\bar{y}_n}=1-\left(\frac{\partial\bar{y}_n}{\partial y_n}\right)^{-1};\tag{D.1}$$

$$\sum_i\left(q^s_i-q_i\right)\frac{\partial x^r_{in}}{\partial z_n}=\left(\frac{\partial\bar{y}_n}{\partial y_n}\right)^{-1}\frac{\partial\bar{y}_n}{\partial z_n};\tag{D.2}$$

$$\sum_i\left(q^s_i-q_i\right)\frac{\mathrm{d}x^r_{in}}{\mathrm{d}q_j}+\sum_i x^r_{in}\frac{\partial q^s_i}{\partial q_j}-x^r_{jn}=\frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_j}.\tag{D.3}$$

These relationships could also be expressed in derivatives of x_{in}^s as:

$$\sum_i (q_i^s - q_i) \frac{\partial x_{in}^s}{\partial y_n} = \frac{\partial \bar{y}_n}{\partial y_n} - 1; \tag{D.4}$$

$$\sum_i (q_i^s - q_i) \frac{\partial x_{in}^s}{\partial z_n} = \frac{\partial \bar{y}_n}{\partial z_n}; \tag{D.5}$$

$$\sum_i (q_i^s - q_i) \frac{\partial x_{in}^s}{\partial q_j} + \sum_i x_{in}^s \frac{\mathrm{d} q_i^s}{\mathrm{d} q_j} - x_{jn}^s = \frac{\mathrm{d} \bar{y}_n}{\mathrm{d} q_j}. \tag{D.6}$$

□

D.2 Proof of lemma 2

Proof. Using (A.7) and (A.12), we transform partial derivative of e_n^s on v_n^s in (A.10) into

$$\frac{\partial e_n^s}{\partial v_n^s} = \frac{1}{u_c} \left[1 + \sum_i (q_i - q_i^s) \frac{\partial x_n^r (q^s, \bar{y}_n, z_n/n)}{\partial \bar{y}_n} \right] = \frac{1}{u_c} \left(\frac{\partial \bar{y}_n}{\partial y_n} \right)^{-1} = \left(\frac{\partial v_n^s}{\partial y_n} \right)^{-1}.$$

The last step is due to relationship between v_n^s and v_n^r in (4).

Use (A.6) and (A.12) we could transform partial derivative of e_n^s on z_n in (A.9) into

$$\frac{\partial e_n^s}{\partial z_n} = -\frac{u_l/n}{u_c} \left[1 + \sum_i (q_i - q_i^s) \frac{\partial x_n^r}{\partial \bar{y}_n} \right] + \sum_i (q_i - q_i^s) \left(\frac{\partial x_n^r}{\partial z_n} \right).$$

Use (D.1) and (D.2) to further simplify it into

$$\frac{\partial e_n^s}{\partial z_n} = -\frac{u_l/n}{u_c} \left(\frac{\partial \bar{y}_n}{\partial y_n} \right)^{-1} - \left(\frac{\partial \bar{y}_n}{\partial y_n} \right)^{-1} \frac{\partial \bar{y}_n}{\partial z_n} = Q_n^s.$$

The last step is due to the first-order condition of first-stage optimization in (A.14).

From (A.8) we infer that $\sum_i q_i^s \frac{\partial x_{in}^{r*}}{\partial q_j^s} + \frac{\partial c_n^{r*}}{\partial q_j^s} = 0$. Therefore, we could transform (A.11) into

$$\frac{\partial e_n^s}{\partial q_j} = \sum_i (q_i - q_i^s) \sum_k \frac{\partial x_{in}^{r*} (q^s, v_n^s, z_n/n)}{\partial q_k^s} \frac{\partial q_k^s}{\partial q_j} + x_{jn}^{r*}. \tag{D.7}$$

Alternative expression of $\frac{\partial e_n^s}{\partial q_j}$ is derived as follows:

$$\begin{aligned}
\frac{\partial e_n^s}{\partial q_j} &= \sum_i (q_i - q_i^s)\left(\frac{\partial x_{in}^r}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n}\sum_k \frac{\partial q_k^s}{\partial q_j} x_{kn}^r\right) + x_{jn}^{r*} \\
&= \sum_i (q_i - q_i^s)\left(\frac{\partial x_{in}^s}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n}\left(\sum_k \frac{\partial q_k^s}{\partial q_j} x_{kn}^r - \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_j}\right)\right) + x_{jn}^{r*} \\
&= \sum_i (q_i - q_i^s)\left(\frac{\partial x_{in}^s}{\partial q_j} - \frac{\partial x_{in}^r}{\partial \bar{y}_n}\sum_k (q_k^s - q_k)\frac{\partial x_{kn}^s}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} x_{jn}^s\right) + x_{jn}^{r*} \\
&= \sum_i (q_i - q_i^s)\left(\frac{\partial x_{in}^s}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} x_{jn}^s\right) - \sum_k (q_k^s - q_k)\frac{\partial x_{kn}^s}{\partial q_j}\sum_i \frac{\partial x_{in}^r}{\partial \bar{y}_n}(q_i - q_i^s) + x_{jn}^{r*} \\
&= \sum_i (q_i - q_i^s)\left(\frac{\partial x_{in}^s}{\partial q_j} + \frac{\partial x_{in}^r}{\partial \bar{y}_n} x_{jn}^s\right) + \sum_i (q_i^s - q_i)\frac{\partial x_{in}^s}{\partial q_j}\left(1 - \left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}\right) + x_{jn}^{r*} \\
&= \left(x_{jn}^s - \sum_i (q_i^s - q_i)\frac{\partial x_{in}^s}{\partial q_j}\right)\left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}.
\end{aligned} \tag{D.8}$$

The first line is obtained from (D.7) by applying (A.12). The second and third line are obtained by using (A.3) and (D.6) separately. The fifth line is derived by using (D.1). □

D.3 Proof of lemma 3

Proof. Roy's identity captures the relationship between $\frac{\partial v_n^s}{\partial q_j}$ and $\frac{\partial v_n^s}{\partial y_n}$. Use (A.5) and (A.4) to transform $\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n}$ into

$$\begin{aligned}
\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n} &= \left(\sum_k \frac{\partial v_n^r}{\partial q_k^s}\frac{\partial q_k^s}{\partial q_i} + \frac{\partial v_n^r}{\partial \bar{y}_n}\frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_i}\right)/\frac{\partial v_n^r}{\partial \bar{y}_n}\frac{\partial \bar{y}_n}{\partial y_n} \\
&= -\left(\sum_k \frac{\partial q_k^s}{\partial q_j} x_{kn}^r - \frac{\mathrm{d}\bar{y}_n}{\mathrm{d}q_j}\right)\left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}.
\end{aligned}$$

Use (D.6) to get an alternative expression as:

$$\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n} = \left[\sum_i (q_i^s - q_i)\frac{\partial x_{in}^s}{\partial q_j} - x_{jn}^s\right]\left(\frac{\partial \bar{y}_n}{\partial y_n}\right)^{-1}.$$

The right-hand side is just the expression of $\frac{\partial e_n^s}{\partial q_j}$ in lemma (D.8) of opposite sign. Therefore, we have

$$\frac{\partial e_n^s}{\partial q_j} = -\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n}, \tag{D.9}$$

as well as

$$\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n} = -x_{jn}^s + \sum_i (q_i^s - q_i) \sum_k \frac{\partial q_k^s}{\partial q_j} \frac{\partial x_{in}^{r*}}{\partial q_k^s}$$

by applying the third equation in lemma 2. Take partial derivatives on z_n and y_n, we get the additional properties about e_{q_j}

$$v_{zq_j} + e_{q_i} v_{yz} + \frac{\mathrm{d}e_{q_i}}{\mathrm{d}z} v_y^s = 0; v_{yq_j} + e_{q_i} v_{yy} + \frac{\partial e_{q_j}}{\partial v} \left(v_y^s\right)^2 = 0. \tag{D.10}$$

To get modified Slutsky equation, using (A.12) to change the following expression

$$\frac{\partial x_{in}^s}{\partial q_j} = \frac{\partial x_{in}^{r*} (q^s, v_n^s, z_n/n)}{\partial q_j} + \frac{\partial x_{in}^{r*} (q^s, v_n^s (q, y_n, z_n/n), z_n/n)}{\partial v_n^s} \frac{\partial v_n^s}{\partial q_i}$$

into

$$\frac{\partial x_{in}^s}{\partial q_j} = \sum_k \frac{\partial x_{in}^{r*} (q^s, v_n^r, z_n/n)}{\partial q_i^s} \frac{\partial q_k^s}{\partial q_j} + \frac{\partial x_{in}^s}{\partial y_n} \left(\frac{\partial v_n^s}{\partial q_j} / \frac{\partial v_n^s}{\partial y_n} \right).$$

□

D.4 Proof of lemma 4

Proof. Using the third equation in (A.17) to substitute $\frac{\mathrm{d}\bar{R}_n}{\mathrm{d}Q_n}$ in (A.16), we get

$$\frac{\mathrm{d}z_n}{\mathrm{d}Q_n} = \frac{\partial z_n^r}{\partial Q_n^s} \frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} \left(1 - (Q_n^s - Q_n) \frac{\partial z_n^r}{\partial \bar{R}_n} \frac{\partial \bar{R}_n}{\partial R_n}\right) - z_n \frac{\partial z_n^r}{\partial \bar{R}_n} \left(\frac{\mathrm{d}Q_n^s}{\mathrm{d}Q_n} - 1\right) \frac{\partial \bar{R}_n}{\partial R_n}.$$

Applying the first equation in (A.17) to substitute items in the first pair of brackets with $\frac{\partial \bar{R}_n}{\partial R_n}$, we could arrive at the expression of $\frac{\mathrm{d}z_n}{\mathrm{d}Q_n}$ in lemma 4. □

D.5 Proof of proposition 1 and equation (19)

Proof. **First-order incentive constraint.** The first-order condition of (17) together with $\dot{y}_n^s =$

$\dot{z}_n \partial \left(z_n - T^s(z_n)\right) / \partial z_n = Q_n^s \dot{z}_n$ yields

$$u_c \left(\frac{\partial c_n^s}{\partial y_n^s} Q_n^s \dot{z}_n + \frac{1}{n} \frac{\partial c_n^s}{\partial l_n} \dot{z}_n \right) + \sum_i u_{x_i} \left(\frac{\partial x_{in}^s}{\partial y_n^s} Q_n^s \dot{z}_n + \frac{1}{n} \frac{\partial x_{in}^s}{\partial l_n} \dot{z}_n \right) + \frac{1}{n} u_l \dot{z}_n = 0. \tag{D.11}$$

Take total derivatives of u_n with respect to n to get

$$\begin{aligned} \dot{u}_n = & u_c \left(\frac{\partial c_n^s}{\partial y_n} \dot{y}_n + \frac{1}{n} \frac{\partial c_n^s}{\partial l_n} \dot{z}_n - \frac{z_n}{n^2} \frac{\partial c_n^s}{\partial l_n} \right) + \sum_i u_{x_i} \left(\frac{\partial x_{in}^s}{\partial y_n} \dot{y}_n + \frac{1}{n} \frac{\partial x_{in}^s}{\partial l_n} \dot{z}_n - \frac{z_n}{n^2} \frac{\partial x_{in}^s}{\partial l_n} \right) \\ & + \frac{1}{n} u_l \dot{z}_n - \frac{z_n}{n^2} u_l, \end{aligned} \tag{D.12}$$

in which $\dot{y}_n = Q_n \dot{z}_n$. We minus (D.12) from (D.11) to get

$$\dot{u}_n = u_c \left(\frac{\partial c_n^s}{\partial y_n} + \sum_i q_i^s \frac{\partial x_{in}^s}{\partial y_n} \right) \left(Q_n - Q_n^s\right) \dot{z}_n - u_c \frac{z_n}{n^2} \left(\frac{\partial c_n^s}{\partial l_n} + \sum_i q_i^s \frac{\partial x_{in}^s}{\partial l_n} \right) - \frac{z_n}{n^2} u_l. \tag{D.13}$$

Using properties of conditional demand function in (A.1) and properties of virtual disposable income in lemma 1, we have

$$\dot{u}_n = -\frac{z_n}{n} \left(\frac{\partial \bar{y}_n}{\partial z_n} u_c + \frac{1}{n} u_l \right) + \left(Q_n - Q_n^s\right) \frac{\partial \bar{y}_n}{\partial y_n} u_c \dot{z}_n.$$

Since we have

$$\frac{\partial v_n^s}{\partial z_n} = \frac{\partial v_n^r}{\partial \bar{y}_n} \frac{\partial \bar{y}_n}{\partial z_n} + \frac{\partial v_n^r}{\partial z_n} = u_c \frac{\partial \bar{y}_n}{\partial z_n} + \frac{1}{n} u_l \tag{D.14}$$

from properties of conditional indirect utility function in (A.4) and (A.5), we could transform (D.14) into

$$\dot{u}_n = u_c \left[\left(Q_n - Q_n^s\right) \dot{z}_n + \frac{z}{n} Q_n^s \right] \frac{\partial \bar{y}_n}{\partial y_n},$$

or

$$\dot{v}_n^s = -\frac{\partial v_n^s}{\partial z_n} \left(\frac{Q_n - Q_n^s}{Q_n^s} \dot{z}_n + \frac{z_n}{n} \right)$$

as in proposition 1.

Spence–Mirrlees and monotonicity condition. Notice that the first-order incentive condition is just a sufficient condition for maximization problem described in (17). We next derive its second-order condition. Denote C_1 the first-order derivative of $u(c_n^s(y_{\tilde{n}}^s, z_{\tilde{n}}/n), x_n^s(y_{\tilde{n}}^s, z_{\tilde{n}}/n), z_{\tilde{n}}/n))$

on $\tilde{n}$, and C_2 the second-order derivative on $\tilde{n}$. Denote C_3 the residual if we subtract C_1 from $\dot{u}_n$. From (D.11) and (D.12) we find the relationship between $\dot{u}_n$ and C_1 as

$$\dot{u}_n = C_1 + C_3,$$

in which

$$C_3 \equiv -\frac{z_n}{n}\left(\frac{\partial \bar{y}_n}{\partial z_n}u_c + \frac{1}{n}u_l\right) + (Q_n - Q_n^s)\frac{\partial \bar{y}_n}{\partial y_n}u_c \dot{z}_n.$$

The first-order condition of maximization problem described in (17) requires that $C_1 = 0$, hence $C_3 = \dot{u}_n$. Denote the total derivative of $\dot{u}_n$ on n by $\ddot{u}_n$, since $dC_3/dn = \ddot{u}_n$, we have

$$\ddot{u}_n = C_2 - \frac{z}{n}\frac{\partial\left(C_1/\dot{z}_n\right)}{\partial z}\dot{z}_n + \ddot{u}_n.$$

Therefore, the second-order condition, which requires that $C_2 \leq 0$, is equivalent to

$$\frac{z}{n}\frac{\partial\left(C_1/\dot{z}_n\right)}{\partial z}\dot{z}_n \leq 0.$$

Since we could use properties of consumer's stage 2 decision to get expression of $C_1/\dot{z}_n$ as $C_1/\dot{z}_n = \partial v_n^s/\partial z_n + Q_n^s \partial v_n^s/\partial y_n$, we could transform second-order condition into

$$C_2 = \frac{z_n}{n}\left(v_{zz}^s - \frac{v_z^s}{v_y^s}v_{yz}^s\right)\dot{z}_n = \frac{z_n v_y^s}{n}\frac{\partial\left(v_z^s/v_y^s\right)}{\partial z_n}\dot{z}_n \leq 0.$$

As long as $v_y^s > 0$ and $\dot{z}_n > 0$, which are not very strict conditions with misperception, we have

$$\frac{\partial\left(v_z^s/v_y^s\right)}{\partial z_n} \leq 0$$

to ensure the first-order condition lead to a maximization optimum. □

D.6 Proof of proposition 2

Proof. The first-order condition of (22) on q_j is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial q_i} =& -\mu\int_N\left[\sum_k \frac{\partial q_k^s}{\partial q_i}\frac{\partial c_n^{r*}(q^s, v_n^s, z_n/n)}{\partial q_k^s} + \sum_k \frac{\partial q_k^s}{\partial q_i}\frac{\partial x_n^{r*}(q^s, v_n^s, z_n/n)}{\partial q_k^s}\right] f(n)\mathrm{d}n \\ &+ \int_N\left[\theta_n\left(v_{zq_i}^s + v_{zy}^s\frac{\partial e}{\partial q_i}\right)\left(\frac{Q_n - Q_n^s}{Q_n^s}\dot{z}_n + \frac{z_n}{n}\right)\right]\mathrm{d}n + \int_N\left[\theta_n v_z^s\left(-\frac{Q_n}{Q_n^s}\frac{1}{Q_n^s}\frac{\partial Q_n^s}{\partial q_j}\dot{z}_n\right)\right]\mathrm{d}n = 0,\end{aligned} \tag{D.15}$$

which could be written as

$$\begin{aligned}&-\int_N\left[\sum_k \frac{\partial q_k^s}{\partial q_i}\frac{\partial c_n^{r*}(q^s, v_n^s, z_n/n)}{\partial q_k^s} + \sum_k \frac{\partial q_k^s}{\partial q_i}\frac{\partial x_n^{r*}(q^s, v_n^s, z_n/n)}{\partial q_k^s}\right] f(n)\mathrm{d}n = \\ &-\int_N\left[\frac{\theta_n}{\mu}\left(v_{zq_i}^s + v_{zy}^s\frac{\partial e}{\partial q_i}\right)\left(\frac{Q_n - Q_n^s}{Q_n^s}\dot{z}_n + \frac{z_n}{n}\right)\right]\mathrm{d}n - \int_N\left(\frac{\theta_n v_z^s}{\mu}\frac{Q_n}{Q_n^s}\frac{1}{Q_n^s}\frac{\partial Q_n^s}{\partial q_j}\dot{z}_n\right)\mathrm{d}n.\end{aligned} \tag{D.16}$$

The item $\frac{\partial c_n^{r*}}{\partial q_k^s}$ in the left hand side of (D.16) could be eliminated using (A.8). As for the right hand side, since (D.9) holds for any $(y_n, z_n) \in R_+^2$, we could take partial derivatives of both sides on z_n to get

$$v_{zq_j}^s + e_{q_j} v_{zy}^s = -v_y^s\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n}.$$

From the definition of misperception wedge on commodity price in (23), we could transform (D.7) into $\frac{\partial e_n}{\partial q_j} = w_{jn}^q + x_{jn}^s$. Therefore, $\frac{de_{q_j}}{dz_n} = \frac{\partial w_{jn}^q}{\partial z_n} + \frac{\partial x_{jn}^s}{\partial z_n}$. Using these conditions, we could simplify (D.16) to be

$$\begin{aligned}&\int_N\sum_i t_i\sum_k\frac{\partial q_k^s}{\partial q_j}\frac{\partial x_{in}^{r*}}{\partial q_k^s} f(n)\mathrm{d}n + \int_N w_{jn}^q f(n)\mathrm{d}n \\ =& \int_N\left[\Theta_n\frac{z_n}{n}\left(\frac{\partial w_{jn}^q}{\partial z_n} + \frac{\partial x_{jn}^s}{\partial z_n}\right)\left(\frac{Q_n - Q_n^s}{Q_n^s}\varepsilon_{zn} + 1\right)\right]\mathrm{d}n - \int_N\left[\Theta_n\frac{z_n}{n}\frac{Q_n}{Q_n^s}\frac{\partial Q_n^s}{\partial q_j}\varepsilon_{zn}\right]\mathrm{d}n\end{aligned} \tag{D.17}$$

, in which $t_i = q_i - 1$ since we have normalized pre-tax commodity price to unity, and $\Theta_n \equiv \theta_n v_y^s/\mu$. □

D.7 Proof of proposition 3

The proof of optimal linear commodity tax formula derived by tax perturbation method is embodied in the first part of appendix C.

D.8 Proof of proposition 4

Proof. It is straightforward to transform (28) using $\frac{\partial x_{in}^{r*}}{\partial z_n} = \frac{\partial x_{in}^{s}}{\partial y_n} Q_n^s + \frac{\partial x_{in}^{s}}{\partial z_n}$ and $e_{q_j} = w_{jn}^q + x_{jn}^s$ into

$$
\begin{aligned}
\int_N \sum_i t_i \left(\sum_k \frac{\partial x_{in}^{r*}}{\partial q_i^s} \frac{\partial q_k^s}{\partial q_j} \right) f(n)\mathrm{d}n - \int_N w_{jn}^q f(n)\mathrm{d}n \\
= - \int_N \left(1 - g_n - \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} \right) f\left(n\right) e_{q_j} dn \\
- \int_N \left(\frac{1 - Q_n}{Q_n} - g_n \tau_n^b + \sum_i \frac{t_i}{Q_n} \frac{x_{in}^s}{z_n} \varepsilon_{x_i z} \right) Q_n \frac{\mathrm{d}z_n}{\mathrm{d}q_j} f(n)\mathrm{d}n.
\end{aligned}
\tag{D.18}
$$

The first item on the right hand side could be found in the expression of Θ_n and the second item of the right hand side relates to the left hand side of optimal income tax formula. Thus we could transform the right hand side of (D.18) into

$$
\begin{aligned}
&\int_N e_{q_j} \mathrm{d}\Theta_n - \int_N \Theta_n \frac{1}{z_n} \frac{\varepsilon_z^I}{\varepsilon_{zQ}^* / \left(\varepsilon_{Qn}^{Qs} + \tilde{\varepsilon}_{Qn}^{Qs} \right)} \frac{\mathrm{d}z_n}{\mathrm{d}n} e_{q_j} \mathrm{d}n \\
&+ \int_N \Theta_n \frac{\mathrm{d}z_n}{\mathrm{d}n} \left(\frac{Q_n}{z_n} \frac{-1}{\varepsilon_{zQ}^* / \left(\varepsilon_{Qn}^{Qs} + \tilde{\varepsilon}_{Qn}^{Qs} \right)} \right) \frac{\mathrm{d}z_n}{\mathrm{d}q_j} \mathrm{d}n \\
&+ \int_N \left[Q'(z_n) \int_N \left(\Theta_{\tilde{n}} \frac{Q_{\tilde{n}}}{Q_{\tilde{n}}^s} \frac{\partial Q_{\tilde{n}}^s}{\partial Q_n} \frac{\mathrm{d}\tilde{z}}{\mathrm{d}\tilde{n}} \right) \mathrm{d}\tilde{n} \right] \frac{\mathrm{d}z_n}{\mathrm{d}q_j} \mathrm{d}n - \int_N \left(\Theta_n \frac{Q'(z_n)}{Q_n} \tilde{\varepsilon}_{Qn}^{Qs} \frac{\mathrm{d}z}{\mathrm{d}n} \right) \frac{\mathrm{d}z_n}{\mathrm{d}q_j} \mathrm{d}n.
\end{aligned}
\tag{D.19}
$$

Then we perform the following transformations on separate parts of (D.19). Firstly, since

$$
\int_N e_{q_j} \mathrm{d}\Theta_n = - \int_N \Theta_n \frac{\mathrm{d}e_{q_j}}{\mathrm{d}n} \mathrm{d}n = \int_N \Theta_n \frac{z}{n} \frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n} \mathrm{d}n - \int_N \Theta_n \left(\frac{\partial e_{q_j}}{\partial v} v_z + \frac{\partial e_{q_j}}{\partial z} + \frac{\partial e_{q_j}}{\partial v} v_y Q_n \right) \frac{\mathrm{d}z}{\mathrm{d}n} \mathrm{d}n,
$$

and income elasticities could be expressed with derivatives of indirect utility function as in the main text, we have

$$
\begin{aligned}
\int_N e_{q_j} \mathrm{d}\Theta_n - \int_N \Theta_n \frac{1}{z_n} \frac{\varepsilon_z^I}{\varepsilon_{zQ}^* / \left(\varepsilon_{Qn}^{Qs} + \tilde{\varepsilon}_{Qn}^{Qs} \right)} \frac{\mathrm{d}z_n}{\mathrm{d}n} e_{q_j} \mathrm{d}n \\
= \int_N \Theta_n \frac{z_n}{n} \frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n} dn - \int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n} \left(\frac{\mathrm{d}e_{q_i}}{\mathrm{d}z_n} + \frac{\partial e_{q_j}}{\partial v_n} v_y Q_n \right) \mathrm{d}n \\
- \int_N \Theta_n \frac{\mathrm{d}z_n}{\mathrm{d}n} \frac{Q}{Q_n^s} \frac{Q_n^s v_{yy}^s + v_{zy}^s}{v_y^s} e_{q_j} \mathrm{d}n.
\end{aligned}
\tag{D.20}
$$

Next, using relationship between elasticities defined using shift function method and in Saez's form as in (B.6), combining with expression of $\mathrm{d}z_n/\mathrm{d}q_j$ in (C.1), we have

$$
\begin{aligned}
&\int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\left(\frac{Q_n}{z}\frac{-1}{\varepsilon^*_{zQ}/\left(\varepsilon^{Qs}_{Qn}+\tilde{\varepsilon}^{Qs}_{Qn}\right)}\right)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\mathrm{d}n \\
=&\int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\left(\frac{Q_n}{z}\frac{-1}{\xi^{cs}_{zQ}/\left(\varepsilon^{Qs}_{Qn}+\tilde{\varepsilon}^{Qs}_{Qn}\right)}\right)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}\left(1+\xi^{cs}_{zQ}\frac{z_n}{Q_n}T''(z_n)\right)\mathrm{d}n \\
=&-\int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\left(\frac{\partial z_n/\partial q_j}{\partial z^*_n/\partial Q_n/\left(\varepsilon^{Qs}_{Qn}+\tilde{\varepsilon}^{Qs}_{Qn}\right)}\right)\mathrm{d}n \\
&-\int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\left[\frac{Q_n}{z}\frac{1}{\xi^{cs}_{zQ}/\left(\varepsilon^{Qs}_{Qn}+\tilde{\varepsilon}^{Qs}_{Qn}\right)}\right]\left[\int_N \frac{\partial z_n}{\partial Q_m}\frac{\mathrm{d}Q_m}{\mathrm{d}q_j}\left(1-\delta_n\right)\mathrm{d}m\right]\mathrm{d}n.
\end{aligned}
\tag{D.21}
$$

Moreover, we change the order of double integral in the forth item of (D.19) to get

$$
\int_N\left[Q'(z_n)\int_N\left(\Theta_{\tilde{n}}\frac{Q_{\tilde{n}}}{Q^s_{\tilde{n}}}\frac{\partial Q^s_{\tilde{n}}}{\partial Q_n}\frac{d\tilde{z}}{d\tilde{n}}\right)\mathrm{d}\tilde{n}\right]\frac{\mathrm{d}z_n}{\mathrm{d}q_j}dn=\int_N\Theta_m\frac{dz_m}{dm}\frac{Q_m}{Q^s_m}\left[\int_N\left(\frac{\partial Q^s_m}{\partial Q_n}\right)Q'(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}q_j}dn\right]\mathrm{dm}.
$$

Combine all these transformations above, we could reorganize the right hand side of (D.18) into

$$
\begin{aligned}
&\int_N \Theta_n \frac{z_n}{n}\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n}\mathrm{d}n \\
&-\int_N \Theta_n \frac{\mathrm{d}z_n}{\mathrm{d}n}\left[\frac{\mathrm{d}e_{q_i}}{\mathrm{d}z_n}+\frac{\partial e_{q_j}}{\partial v}v^s_y Q_n+\frac{Q_n}{Q^s_n}\left(\frac{Q^s_n v^s_{yy}+v^s_{zy}}{v^s_y}e_{q_j}+\frac{Q^s_n v^s_{yq_j}+v^s_{zq_j}}{v^s_y}+\frac{\partial Q^s_n}{\partial q_j}\right)\right]\mathrm{d}n \\
&-\int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\frac{\varepsilon^{Qs}_{Qn}+\tilde{\varepsilon}^{Qs}_{Qn}}{\partial z^*_n/\partial Q_n}\left[\int_N \frac{\partial z_n}{\partial Q_m}Q'(z_m)\frac{\mathrm{d}z_m}{\mathrm{d}q_j}\left(1-\delta_n\right)\mathrm{d}m\right]\mathrm{d}n \\
&+\int_N \Theta_n \frac{\mathrm{d}z_n}{\mathrm{d}n}\frac{Q_n}{Q^s_n}\left[\int_N \frac{\partial Q^s_n}{\partial Q_m}Q'(z_m)\frac{\mathrm{d}z_m}{\mathrm{d}q_j}\left(1-\delta_n\right)\mathrm{d}m\right]\mathrm{dn}.
\end{aligned}
\tag{D.22}
$$

While we could use properties of e_{q_j} in (D.10), to simplify the first line plus second line of (D.22) into

$$
\int_N \Theta_n \frac{z}{n}\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n}\mathrm{d}n+\int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\left(\frac{Q_n}{Q^s_n}-1\right)\frac{\mathrm{d}e_{q_j}}{\mathrm{d}z_n}\mathrm{d}n-\int_N \Theta_n \frac{\mathrm{d}z}{\mathrm{d}n}\frac{Q_n}{Q^s_n}\frac{\partial Q^s_n}{\partial q_j}\mathrm{d}n.
$$

If we express the above integral equation with integral on z, we exactly get the right hand side of (25). The last two lines of (D.22) cancel out since $\frac{\partial Q^s_n}{\partial Q_m}=\frac{\partial z_n/\partial Q_m}{\partial z^*_n/\partial Q_n}\left(\frac{\partial Q^s_n}{\partial Q_n}+\frac{\partial Q^s_n}{\partial \mathbf{Q}}\cdot\delta_n\right)$. In brief, optimal commodity tax derived from tax perturbation method could be transformed into expression in proposition 2 derived from mechanism design. $\square$

D.9 Proof of proposition 5

Proof. The first-order conditions of government's maximization problem with regard to z_n, v_n^s and κ_n are:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial z_n} =&\mu\left(1-\frac{\partial c_n^{r*}}{\partial z_n}-\frac{\sum_i \partial x_{in}^{r*}}{\partial z_n}\right)f(n)+\theta_n\left(v_{zz}^s+v_{zy}^s\frac{\partial e_n}{\partial z_n}\right)\left(\frac{Q_n-Q_n^s}{Q_n^s}\kappa_n+\frac{z_n}{n}\right)+\frac{\theta_n}{n}v_z^s\\ &+\theta_n v_z^s\frac{Q'(z_n)}{Q_n^s}\left(1-\frac{Q_n}{Q_n^s}\frac{\partial Q_n^s}{\partial Q_n}\right)\kappa_n-Q'(z_n)\int_N\left(\theta_{\tilde{n}}v_{\tilde{z}}\frac{Q_{\tilde{n}}}{(Q_{\tilde{n}}^s)^2}\frac{\partial Q_{\tilde{n}}^s}{\partial Q_n}\kappa_{\tilde{n}}\right)\mathrm{d}\tilde{n}-\dot{\lambda}_n=0; \quad (D.23)\\ \frac{\partial \mathcal{L}}{\partial v_n^s} =&\left(\Psi'\left(v_n^s\right)-\mu\frac{\partial c_n^{r*}}{\partial v_n^s}-\mu\frac{\partial \sum x_n^{r*}}{\partial v_n^s}\right)f(n)+\theta_n v_{zy}^s\frac{\partial e_n}{\partial v_n^s}\left(\frac{Q_n-Q_n^s}{Q_n^s}\dot{z}_n+\frac{z_n}{n}\right)-\dot{\theta}_n=0; \quad (D.24)\\ \frac{\partial \mathcal{L}}{\partial \kappa_n} =&\theta_n\frac{\partial v_n^s\left(q,e_n\left(q,v_n^s,z_n/n\right),z_n/n\right)}{\partial z_n}\left(\frac{Q_n-Q_n^s}{Q_n^s}\right)-\lambda_n=0. \quad (D.25)\end{aligned}$$

We first eliminate $\dot{\lambda}$ in (D.23) with (D.25) and (D.24), then use elasticities defined in the main paper to yo simplify the first-order condition on z_n.

Take total derivatives with respect to n on both sides of (D.25), and use (D.24) to substitute $\dot{\theta}_n$ to get expression of $\dot{\lambda}_n$ as

$$\begin{aligned}\dot{\lambda}_n=&\left(\Psi'\left(v_n^s\right)-\mu\frac{\partial c_n^{r*}}{\partial v_n^s}-\mu\frac{\partial\sum x_n^{r*}}{\partial v_n^s}\right)v_y^s\left(Q_n^s-Q_n\right)f(n)\\ &+\theta_n\left(Q_n^s-Q_n\right)v_{zy}^s\left(\frac{Q_n-Q_n^s}{Q_n^s}\dot{z}_n+\frac{z_n}{n}\right)\\ &+\theta_n\left(Q_n^s-Q_n\right)\left[\left(v_{yy}^sQ_n+v_{yz}^s\right)\dot{z}_n-\frac{z}{n}v_{yz}^s\right]+\theta_n\frac{\partial v_n^s}{\partial y_n}\dot{z}_n\left(\frac{\partial Q_n^s}{\partial Q_n}+\frac{\partial Q^s}{\partial \mathbf{Q}}\cdot\delta_n-1\right){Q'}_n.\end{aligned} \quad (D.26)$$

Use expression of $\dot{\lambda}$ in (D.26) and definition of D_n in (11) to transform (D.23) into:

$$\begin{aligned}&\left(1-\frac{\partial c_n^{r*}}{\partial z_n}-\frac{\sum_i\partial x_{in}^{r*}}{\partial z_n}\right)-\left(Q_n^s-Q_n\right)v_y^s\left(\Psi'\left(v_n^s\right)/\mu-\frac{\partial c_n^{r*}}{\partial v_n^s}-\frac{\partial\sum x_n^{r*}}{\partial v_n^s}\right)\\ =&-\frac{\theta_n}{n\mu f(n)}\left[\left(v_{zz}^s+v_{zy}^sQ_n^s\right)z_n+v_z^s\right]+\frac{\theta_n}{\mu f(n)}\frac{Q_n^s-Q_n}{Q_n^s}D_n\dot{z}_n\\ &-Q'(z_n)\int_N\left(\frac{\theta_{\tilde{n}}}{\mu f(n)}v_{\tilde{y}}^s\frac{Q_{\tilde{n}}}{Q_{\tilde{n}}^s}\frac{\partial Q_{\tilde{n}}^s}{\partial Q_n}\dot{z}_{\tilde{n}}\right)(1-\delta_n)\mathrm{d}\tilde{n}.\end{aligned}$$

Then eliminate partial derivatives of c_n^{r*} by (A.6) and (A.7). Notice that the second term on the right hand side of equation also appears in numerator of expression of ε_{zn} in (16) , while D_n appears both in the expression of ε_{zn} and expression of ε^*_{zQ} in (12). We could then transform the equation above into

$$\frac{T'(z)}{1-T'(z)}-\frac{\Psi'\left(v^s_n\right)v^s_y}{\mu}\frac{Q^s_n-Q_n}{Q_n}+\sum_i\frac{q_i-1}{Q}\left(\frac{\partial x^s_{in}}{\partial y_n}Q_n+\frac{\partial x^s_{in}}{\partial z_n}\right)$$
$$=-\frac{\theta_n v^s_y}{n\mu f(n)}\frac{\varepsilon_{zn}}{\varepsilon^*_{zQ}/\left(\varepsilon^{Qs}_{Qn}+\hat{\varepsilon}^{Qs}_{Qn}\right)}-Q'(z_n)\int_N\left(\frac{\theta_{\tilde{n}}v^s_{\tilde{y}}}{\mu f(n)}\frac{Q_{\tilde{n}}}{Q^s_{\tilde{n}}}\frac{\partial Q^s_{\tilde{n}}}{\partial Q_n}\dot{z}_{\tilde{n}}\right)(1-\delta_n)\mathrm{d}\tilde{n}$$

Using definitions that $\varepsilon^{Qs}_{Qn}\equiv\frac{Q}{Q^s_n}\frac{\partial Q^s_n}{\partial Q_n};\varepsilon^{\tilde{Q}s}_{Qn}\equiv\frac{Q_{\tilde{n}}}{Q^s_{\tilde{n}}}\frac{\partial Q^s_{\tilde{n}}}{\partial Q_n};\hat{\varepsilon}^{Qs}_{Qn}\equiv\frac{Q_n}{Q^s_n}\frac{\partial Q^s_n}{\partial \mathbf{Q}}\cdot\delta_n;\ \tau^b_n\equiv\frac{Q^s_n-Q_n}{Q_n};\ g_n\equiv\frac{\Psi'(v^s_n)v^s_y}{\mu}$ and $\varepsilon_{x_iz}\equiv\frac{z_n}{x^s_{in}}\left(\frac{\partial x^s_{in}}{\partial y_n}Q_n+\frac{\partial x^s_{in}}{\partial z_n}\right)$, the above equation could be expressed with behavioral elasticities as

$$\begin{aligned}\frac{1-Q_n}{Q_n}-g_n\tau^b_n+\sum_i\frac{t_i}{Q_n}\frac{x^s_{in}}{z_n}\varepsilon_{x_iz}=&-\frac{\Theta_n}{nf(n)}\frac{\varepsilon_{zn}}{\varepsilon^*_{zQ}/\left(\varepsilon^{Qs}_{Qn}+\hat{\varepsilon}^{Qs}_{Qn}\right)}\\&-\frac{Q'(z_n)}{Q_nf(n)}\int_N\left(\Theta_{\tilde{n}}\frac{z_{\tilde{n}}}{\tilde{n}}\varepsilon^{\tilde{Q}s}_{Qn}\varepsilon_{z\tilde{n}}\right)(1-\delta_n)\mathrm{d}\tilde{n},\end{aligned}\tag{D.27}$$

in which $\Theta_n\equiv\theta_n v^s_y/\mu$ To get expression of $\Theta_n\Theta_n$, take full derivatives of $\theta_n v^s_y/\mu$ with respect to n:

$$\dot{\Theta}_n=\frac{\theta_n}{\mu}\left[\left(v^s_{yy}Q_n+v^s_{yz}\right)\dot{z}_n-\frac{z}{n}v^s_{yz}\right]+v^s_y\dot{\theta}_n/\mu.$$

Use (D.24) to eliminate $\dot{\theta}$ and get

$$\dot{\Theta}_n=-\left(1-g_n-\sum_i\left(q_i-1\right)\frac{\partial x^s_{in}}{\partial y_n}\right)f(n)+\Theta_n\frac{1}{n}\frac{\varepsilon^I_z}{\varepsilon^*_{zQ}/\left(\varepsilon^{Qs}_{Qn}+\hat{\varepsilon}^{Qs}_{Qn}\right)}\varepsilon_{zn}.$$

The solution to this integral equation is

$$\Theta_n=\int\limits_n^{\bar{n}}e^{-\int\limits_n^{n'}\rho(s)ds}\left(1-g_{n'}-\sum_i\left(q_i-1\right)\frac{\partial x^s_{in'}}{\partial y_{n'}}\right)f(n')dn';\rho=\frac{1}{n}\frac{\varepsilon^I_z}{\varepsilon^*_{zQ}/\left(\varepsilon^{Qs}_{Qn}+\hat{\varepsilon}^{Qs}_{Qn}\right)}\varepsilon_{zn}.$$

Finally, we use $F(n)\equiv\tilde{F}(z_n)$,$nf(n)=\varepsilon_{zn}z_n\tilde{f}(z_n)$ and $\int\limits_n^{\bar{n}}af(n)\mathrm{d}n=\int\limits_z^{\bar{z}}a\tilde{f}(z)\mathrm{d}z$ to transform (D.27) and Θ_n in terms of labor income densities as in (35) and (36). □

D.10 Proof of corollary 1

Proof. We first transform optimal commodity tax formula (28) into

$$
\begin{aligned}
&\int_N \sum_i t_i \left(\sum_k \frac{\partial x_{in}^{r*}}{\partial q_i^s} \frac{\partial q_k^s}{\partial q_j} \right) f(n)\mathrm{d}n - \int_N w_{jn}^q f(n)\mathrm{d}n \\
=& - \int_N \left(1 - g_n - \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} \right) \left(x_{jn}^s + w_{jn}^q \right) f(n)\mathrm{d}n \\
&- \int_N \left(\frac{1 - Q_n}{Q_n} - g_n \tau_n^b + \sum_i \frac{t_i}{Q_n} \frac{x_{in}^s}{z_n} \varepsilon_{x_i z} \right) Q_n \frac{\mathrm{d}z_n}{\mathrm{d}q_j} f(n)\mathrm{d}n.
\end{aligned} \tag{D.28}
$$

Use (D.27) to substitute expression in the brackets in the second item on the right hand side of this formula. Use expression of $\mathrm{d}z_n/\mathrm{d}q_j$ in (C.1) to decompose the influence of q_j on z_n. The right hand side of equation (D.28) could be transformed into

$$
\begin{aligned}
RHS =& - \int_N \left(1 - g_n - \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} \right) \left(x_{jn}^s + w_{jn}^q \right) f(n)\mathrm{d}n \\
&+ \int_N \left(\frac{\Theta_n}{n} \frac{\varepsilon_{zn}}{\varepsilon_{zQ}^* / \left(\varepsilon_{Qn}^{Qs} + \tilde{\varepsilon}_{Qn}^{Qs} \right)} \right) \left(1 + \xi_{zQ}^{cs} \frac{z_n}{Q_n} T''(z_n) \right)^{-1} Q_n \frac{\partial z_n}{\partial q_j} \mathrm{d}n \\
&+ \int_N \frac{\Theta_n}{n} \frac{Q_n \varepsilon_{zn}}{\varepsilon_{zQ}^* / \left(\varepsilon_{Qn}^{Qs} + \tilde{\varepsilon}_{Qn}^{Qs} \right)} \left(1 + \xi_{zQ}^{cs} \frac{z_n}{Q_n} T''(z_n) \right)^{-1} \int_N \frac{\partial z_n}{\partial Q_m} \frac{\mathrm{d}Q_m}{\mathrm{d}q_j} \left(1 - \delta_n \right) \mathrm{d}m\mathrm{d}n \\
&+ \int_N Q'(z_n) \frac{\mathrm{d}z_n}{\mathrm{d}q_j} \int_N \left(\Theta_{\tilde{n}} \frac{z_{\tilde{n}}}{\tilde{n}} \varepsilon_{Qn}^{\tilde{Q}s} \varepsilon_{z\tilde{n}} \right) \left(1 - \delta_n \right) \mathrm{d}\tilde{n}\mathrm{d}n.
\end{aligned} \tag{D.29}
$$

The sum of last two items equals to zero, which could be proved by exchanging integral order in either of the double integral.

Use the expression of elasticities defined in section II, we could decompose $Q_n \frac{\partial z_n}{\partial q_j}$ into

$$
Q_n \frac{\partial z_n}{\partial q_j} = \varepsilon_z^I e_{q_i} + \left(\frac{\partial e_{q_j}}{\partial z} - \frac{\partial Q_n^s}{\partial q_j} \right) \frac{Q_n}{Q_n^s} \frac{z_n \varepsilon_{zQ}^*}{\varepsilon_{Qn}^{Qs} + \tilde{\varepsilon}_{Qn}^{Qs}}. \tag{D.30}
$$

Saez (2002) also makes similar decomposition (in Lemma 1) to compute influence of commodity tax reform on labor income. However, he uses First-order Taylor expansion in his proof, hence our decomposition is more precise taken into consideration the non-linearity of income tax schedule.

Using (D.30), we could transform the right hand side of equation (D.28) for a step further as:

$$
\begin{aligned}
RHS = & -\int_N (1-\gamma_n) e_{q_i} f(n) \mathrm{d}n \\
& -\int_N \frac{z_n \varepsilon_{zn}}{n} \Theta_n \frac{\tilde{f}^*(z_n)}{\tilde{f}(z_n)} \frac{Q_n}{Q_n^s} \frac{\partial Q_n^s}{\partial q_j} \mathrm{d}n.
\end{aligned} \tag{D.31}
$$

As in Saez (2001), $\tilde{f}^*(z_n)$ is virtual density of income distribution. The relationship between $\tilde{f}^*(z_n)$ and $\tilde{f}(z_n)$ satisfies

$$
\frac{h^*(z_n)}{h(z_n)} = \left(1+\xi_{zQ}^{cs} \frac{z_n}{Q_n} T''(z_n)\right)^{-1}.
$$

γ_n is the marginal social utility of one unit of government transfer to consumer n which is defined by

$$
\gamma_n \equiv g_n + \sum_i t_i \frac{\partial x_{in}^s}{\partial y_n} + \frac{h^*(z_n)}{f(n) h(z_n)} \frac{\varepsilon_{zn}\Theta_n}{n} \frac{Q_n}{Q_n^s} \left[\frac{\varepsilon_z^I}{\varepsilon_{zQ}^*} \left(\frac{\partial Q_n^s}{\partial Q_n} + \frac{\partial Q_n^s}{\partial \mathbf{Q}} \cdot \delta_n \right) + \varepsilon_{e_j z} \right].
$$

$\varepsilon_{e_j z}$ is elasticity of e_{q_j} on labor income z_n, which is defined by $\varepsilon_{e_j z} \equiv \frac{z_n}{e_{q_i}} \frac{\partial e_{q_j}(q,v,z_n/n)}{\partial z}$. Then we express (D.28) with integral on z and rearrange it into

$$
\begin{aligned}
-\int_Z \sum_i t_i \left(\sum_k \frac{\partial x_{in}^{r*}}{\partial q_i^s} \frac{\partial q_k^s}{\partial q_j} \right) \tilde{f}(z) \mathrm{d}z = & -\int_Z w_{jn}^q \tilde{f}(z) \mathrm{d}z + \int_Z (1-\gamma_n) e_{q_i} \tilde{f}(z) \mathrm{d}z \\
& + \int_Z \frac{1}{n} \Theta_n \frac{h^*(z_n)}{h(z_n)} \frac{Q_n}{Q_n^s} \frac{\partial Q_n^s}{\partial q_j} \mathrm{d}z.
\end{aligned} \tag{D.32}
$$

It is then straightforward to transform the above equation into

$$
\begin{aligned}
-\frac{1}{\overline{w_j^q} + \overline{x_j^s}} \overline{\sum_i t_i \left(\sum_k \frac{\partial x_i^{r*}}{\partial q_i^s} \frac{\partial q_k^s}{\partial q_j} \right)} = & 1 - \overline{\gamma} - \operatorname{cov}\left(\gamma, \frac{e_{q_i}}{\overline{w_j^q} + \overline{x_j^s}} \right) \\
& - \frac{\overline{w_j^q}}{\overline{w_j^q} + \overline{x_j^s}} + \frac{1}{\overline{w_j^q} + \overline{x_j^s}} \int_N \frac{\Theta_n}{n} \frac{h^*(z_n)}{h(z_n)} \frac{Q_n}{Q_n^s} \frac{\partial Q_n^s}{\partial q_j} \mathrm{d}z.
\end{aligned} \tag{D.33}
$$

The "bar" indicates an integral on z. For example, $\overline{w_j^q} \equiv \int_Z w_{jn}^q \tilde{f}(z) \mathrm{d}z, \overline{x_j^s} \equiv \int_N x_{jn}^s \tilde{f}(z) \mathrm{d}z$. □

D.11 Proof of corollary 2

Proof. From the first-order condition (3) and budget constraint in second stage optimization problem, we find that when individual's utility is weakly separable between commodities and labor, conditional commodity demands x_n^s and c_n^s are independent on z_n so that $\frac{\partial x_{jn}^s}{\partial z_n} = 0$ for $\forall j \in \{1 : I\}$.

Moreover, since $v_n^s(q, y_n, z_n) = u(c_n^s, x_n^s, z_n/n) = u(h(c_n^s, x_n^s), z_n/n)$, we also find that $\frac{\partial v_n^s}{\partial q_j}/\frac{\partial v_n^s}{\partial y_n}$ is irrelevant to z_n. Therefore, in Slustly equation (8), all items except the first item on the right-hand side are irrelevant to z_n, which means $\frac{\partial w_{jn}^q}{\partial z_n} = 0$. Use these conditions to simplify (26) to get (31). □

D.12 Proof of equation (33)

In this part we need to prove that under both homothetic sub-utility function and linear within-group Engel curves lead to the conditional the following form of compensated demand

$$x_{jn}^{r*} = a_j(q) + b_j(q)\, d_n, \forall j. \tag{D.34}$$

Proof. **Case 1.** If sub-utility function of general goods is homothetic, then we could express utility function as $u(c_n, h(x_n), z_n/n)$, in which $h(x)$ is homogeneous of degree one. Then we have the following properties for h as:

$$h_n = \sum_i x_{in}^s h_i.$$

The first-order condition of consumer's second stage maximization problem implies that $h_i/h_j = q_i^s/q_j^s$ for $\forall i, j \in \{1 : I\}$. The budget constraint of an as-if rational consumer still requires that $c_n + \sum_i q_i^s x_{in}^s = \bar{y}_n$. Then we have

$$h_n = \sum_i x_{in}^s h_i = \frac{h_j}{q_j^s} \sum_i x_{in}^s q_i^s = \frac{h_j}{q_j^s}\left(\bar{y}_n - c_n\right),$$

which implies

$$h\left(\frac{x_n^s}{\bar{y}_n - c_n}\right) = \frac{1}{\bar{y}_n - c_n} h_n = \frac{1}{q_j^s} h_j \left(\frac{x_n^s}{\bar{y}_n - c_n}\right).$$

Since budget constraint of as-if rational consumers could be transformed into

$$\sum_i q_i^s \frac{x_{in}^s}{\bar{y}_n - c_n} = 1,$$

we find that $\frac{x_{in}^s}{\bar{y}_n - c_n}$ is only influenced by q^s and is independent of n. As q^s depends on actual price vector q, we could express conditional commodity demand as $x_{jn}^s = b_j(q)\left(\bar{y}_n - c_n^s\right)$ and express conditional compensated commodity demand as

$$x_{jn}^{r*} = b_j\left(q\right)\left(e_n^r - c_n^{r*}\right).$$

Case 2. If within-group Engel curves are linear, then we have $x_{jn}^s = b_j\left(q\right)\left(y_n - c_n^s\right)$. For conditional compensated commodity demand, we could express it as

$$x_{jn}^{r*} = b_j\left(q\right)\left(e_n^s - c_n^{r*}\right).$$

Therefore, compensated commodity demand in both cases satisfy the form in (33). □

D.13 Proof of equation (39) and (40)

Proof. **Optimal commodity tax.** The first-order condition of consumer's second stage utility maximization problem is

$$\frac{1-\beta}{\beta}\frac{c_n^s}{x_n^s} = q^s.$$

Combine it with budget constraint $c_n + qx_n = y_n$ to get conditional demand function and indirect utility function as

$$c_n^s = \frac{q^s}{q^s + \frac{1-\beta}{\beta}q}y_n; x_n^s = \frac{1}{\frac{\beta}{1-\beta}q^s + q}y_n; v_n^s = \beta\ln\left(\frac{\beta}{1-\beta}q^s\right) - \ln\left(\frac{\beta}{1-\beta}q^s + q\right) - \frac{1}{\sigma}\left(\frac{z_n}{n}\right)^\sigma + \ln y_n.$$

As-if rational consumer's expenditure-minimizing problem is

$$\min_{c_n,x_n}\left(c_n + q^s x_n\right), s.t.\beta\ln c_n + \left(1-\beta\right)\ln x_n - \frac{1}{\sigma}\left(\frac{z_n}{n}\right)^\sigma \geq v_n.$$

The solution gives conditional demand function and misperception wedge which satisfy

$$\frac{\partial x_n^{r*}}{\partial q^s} = -\beta\frac{x_n^s}{q^s}; w_n^q\left(q, y, z_n/n\right) = -\beta\frac{q-q^s}{q^s}\frac{\partial q_i^s}{\partial q_i}x_n^s.$$

Put these specifications into (25), we find that optimal t should be set to satisfy

$$\frac{q^s-1}{q^s}\frac{\partial q^s}{\partial q}\left(\frac{1}{1-\beta}q^s + \frac{1}{\beta}q\right)^{-1} = \frac{\int_Z \Theta_z \frac{Q}{Q^s}\frac{\partial Q^s}{\partial q}\mathrm{d}z}{\int_Z\left(z - T\left(z\right)\right)\tilde{f}(z)\mathrm{d}z},$$

in which

$$\Theta_z = (z - T(z)) \int\limits_{z}^{z_{\max}} \frac{1}{z' - T(z')} \left(1 - g - \frac{1}{\frac{\beta}{1-\beta} q^s + q} t \right) \tilde{f}(z') dz'.$$

Optimal income tax. We also have (35) be transformed into

$$\begin{aligned} &\frac{T'(z)}{1 - T'(z)} - g_n \tau_n^b + \frac{t}{\frac{\beta}{1-\beta} q^s + q} \\ &= -\frac{D(z)}{\tilde{f}(z)} \left[\frac{z - T(z)}{Q^s(z)} \Theta_z + \frac{z T''(z)}{1 - T'(z)} \frac{1}{\frac{\sigma}{n} \left(\frac{z}{n} \right)^{\sigma}} \int_Z \left(\frac{\partial Q^s(m)}{\partial Q(z)} \frac{Q(m)}{Q^s(m)} \Theta_m (1 - \delta_z) \right) \mathrm{dm} \right], \end{aligned} \tag{D.35}$$

in which

$$D(z) = \frac{1}{z - T(z)} \left(\frac{\partial Q^s}{\partial Q} + \frac{\partial Q^s}{\partial \mathbf{Q}} \cdot \delta_z \right) Q'(z) - \left(\frac{1}{z - T(z)} \right)^2 Q^s(z) Q(z) - \frac{\sigma - 1}{n^2} \left(\frac{z}{n} \right)^{\sigma - 2}.$$

Use expression of optimal commodity tax to rewrite optimal income tax formula as

$$\begin{aligned} \frac{T'(z)}{1 - T'(z)} - g_n \tau_n^b = &- \frac{D(z)}{\tilde{f}(z)} \frac{z - T(z)}{Q^s(z)} \Theta_z \\ &- \frac{D(z)}{\tilde{f}(z)} \frac{z T''(z)}{1 - T'(z)} \frac{n}{\sigma} \left(\frac{z}{n} \right)^{-\sigma} \int_Z \Theta_m \frac{Q(m)}{Q^s(m)} \frac{\partial Q^s(m)}{\partial Q(z)} (1 - \delta_z) \mathrm{dm} \\ &- \frac{(q-1) q^s}{(q^s - 1) \frac{\partial q^s}{\partial q}} \frac{1}{\beta \int_Z (z - T(z)) \tilde{f}(z) \mathrm{d}z} \int_Z \Theta_{\mathrm{m}} \frac{Q(m)}{Q^s(m)} \frac{\partial Q^s(m)}{\partial q} \mathrm{dm}. \end{aligned}$$

□

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